Inflationary Cosmology: from Theory to Observation and Back

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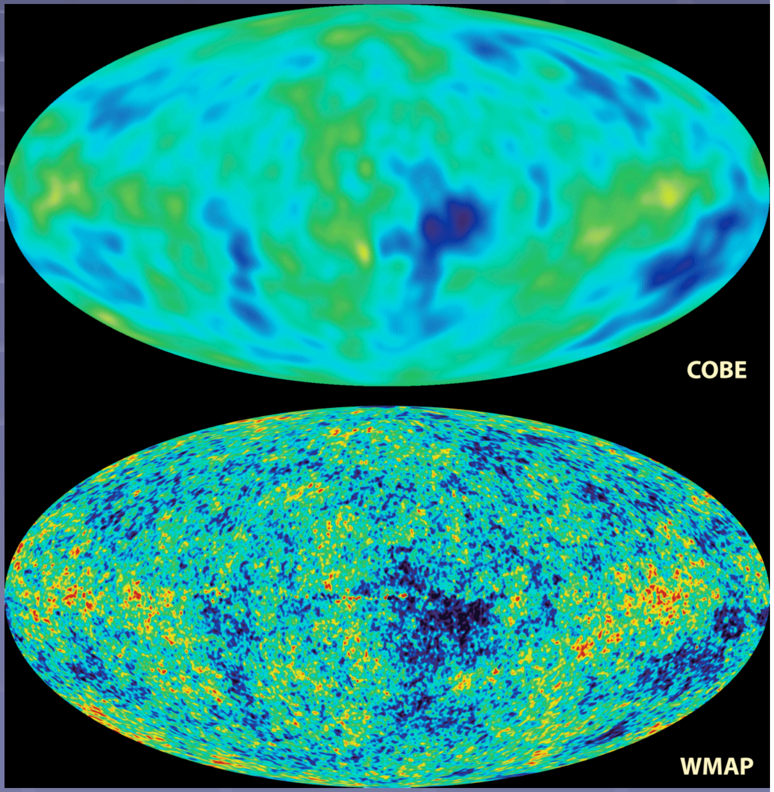


"Big Questions" in Cosmology

- The Hot Big Bang is clearly right, but our understanding is incomplete.
- 1. What is the Dark Matter?
- 2. What is the Dark Energy?
- 3. What is the Inflaton?
- My research has focused on looking for answers to these questions.
- WHAT'S NEW? Today, my talk is about inflation because our theoretical ideas are being tested with new data released by the WMAP team last month.

On: the role of observations

"Faith is a fine invention
When Gentlemen can see ---
But Microscopes are prudent
In an Emergency

Emily Dickinson, 1860

We're starting to see the internal machinery of the mechanism that drives inflation.

OUTLINE

- I. Puzzles Unresolved by Hot Big Bang cosmology
- II. Resolution proposed by Inflationary Scenario
- III Theoretical Models: (i) tunneling (ii) rolling
- IV. Testable Predictions of Inflation
- V. How do these predictions compare to data?
- VI. What do data tell us about theory?

Outline

- Why Inflation?
- Inflation Solves Cosmological Problems
- Theoretical Ideas in Inflation
- Testing Inflation
- Focus on Natural Inflation: theory and data

SUMMARY:

- I. The predictions of inflation are right:
  (i) the universe has a critical density
  (ii) Gaussian perturbations
  (iii) superhorizon fluctuations
  (iv) density perturbation spectrum nearly scale invariant
  (v) gravitational wave modes detectable in upcoming polarization experiments
- II. Polarization measurements will tell us which model is right.
  WMAP already selects between models.
  Natural inflation (Freese, Frieman, Olinto) looks great

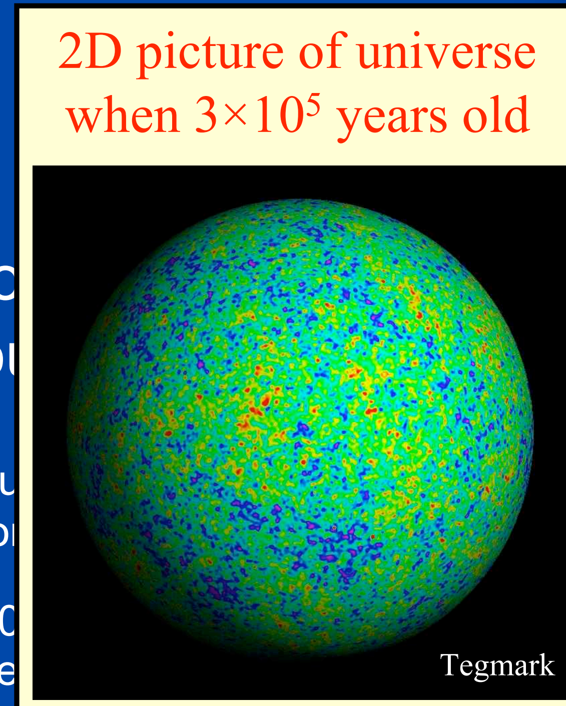
WHY INFLATION?

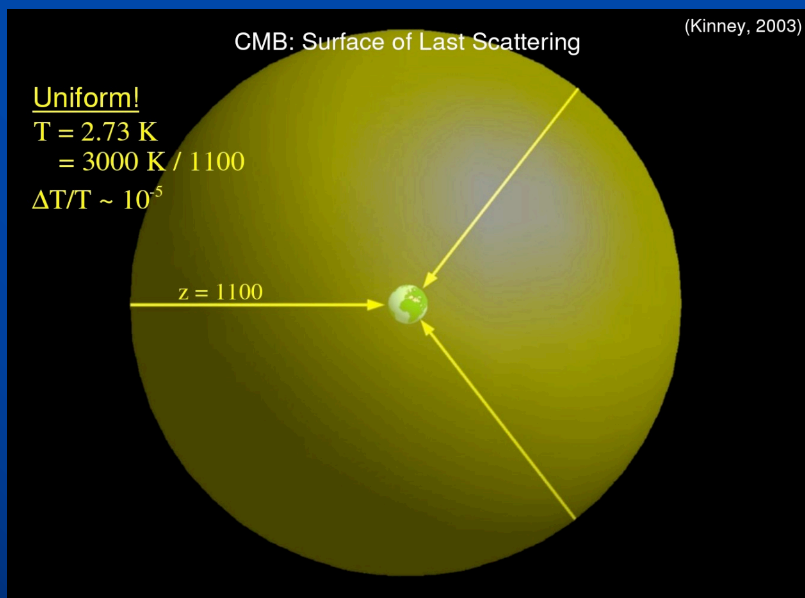
- Cosmological Puzzles unresolved by standard Hot Big Bang:
- 1) Large-scale ‘smoothness’ -- homogeneity and isotropy
- 2) flatness and oldness
- 3) GUT magnetic monopoles

- The idea of inflation was proposed to resolve these puzzles
- BONUS: causal generation of density fluctuations required for galaxy formation

1) The Homogeneity Problem

- Universe is homogenous and isotro
  scales: cosmic microwave backgrou





Early u
(photo

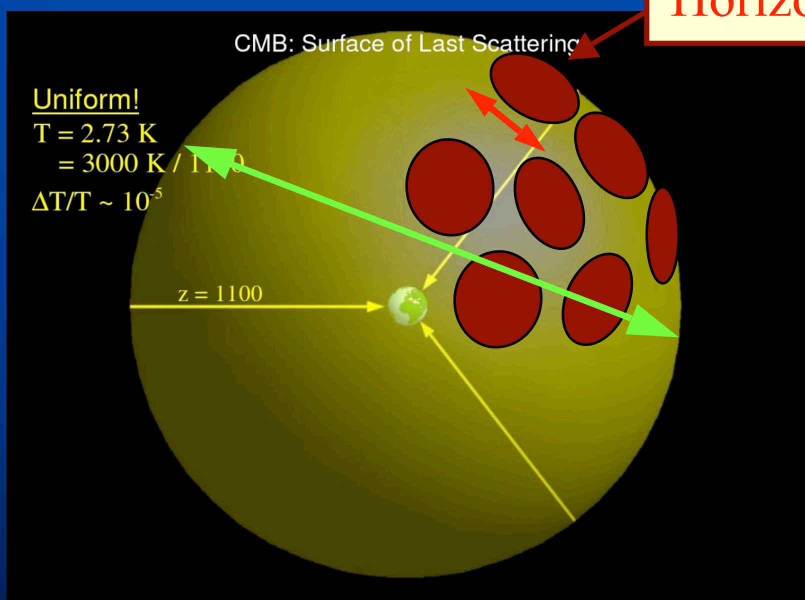
t ~ 300
Ions re
universe becomes transparent to radiation (photon no longer scatter)

Today (T ~ 3 K).
In each direction in the sky, photons arrive that last scattered during recombination. The origin of these photons is the "surface of last scattering."

Homogeneity

- Uniform to $^{\Delta T}/_{T} \sim 10^{-5}$
  - Our observable universe today scaled back to the time of last scattering is $\sim 10^{25}$ cm
  - Horizon size at the time of last scattering is $ct \sim 10^{20}$ cm (corresponds to $\theta \sim 1^{o}$)

Horizon ct



Present observable universe was then $\sim 10^{5}$ causally distinct regions!

2) The Flatness Problem: Curvature

- Einstein's equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

ρ : energy density
$\propto a^{-4}$ (radiation)
$\propto a^{-3}$ (matter)

a : scale factor
of the universe

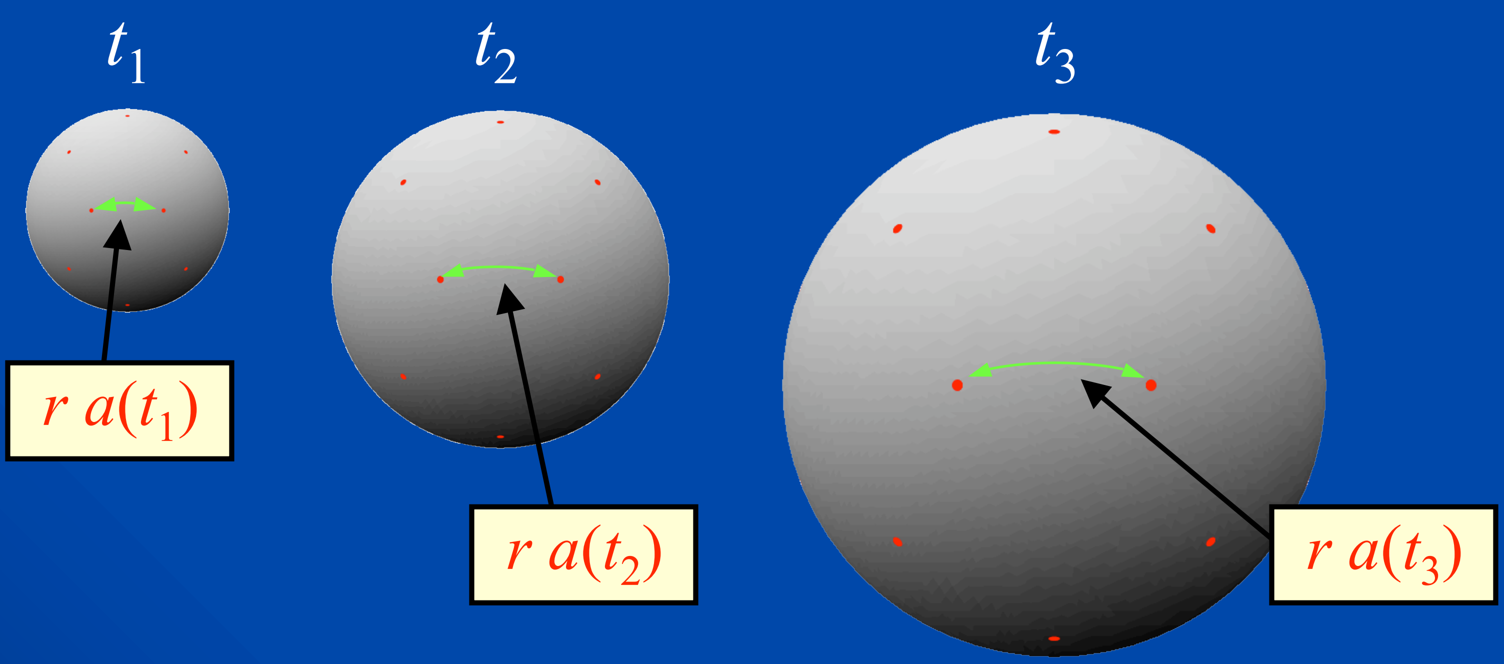
curvature $k = 0, \pm 1$

- Critical density

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h_0^2 \ \mathrm{g/cm^3} \ \sim 10^{-46}\ \mathrm{GeV}^4$$

$$\Omega \equiv \frac{\rho}{\rho_c}$$

Expansion



- Hubble constant

$$H_0 = \frac{\dot{a}}{a} = 100\, h_0 \,\frac{\text{km/s}}{\text{Mpc}}$$

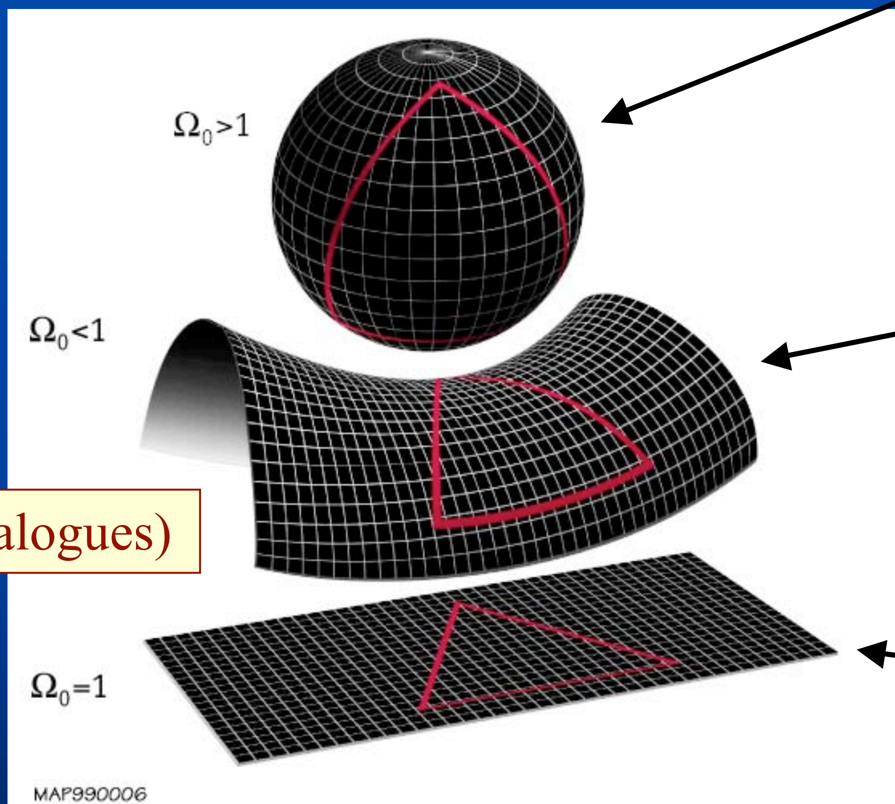
$$h_0 \sim 0.7$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

$$\Omega \equiv \frac{\rho}{\rho_c}$$

Curvature

- 3 types of models



NASA/WMAP Science Team

(2D analogues)

Closed ($k < 0$):
- positively curved
- $\Omega > 1$

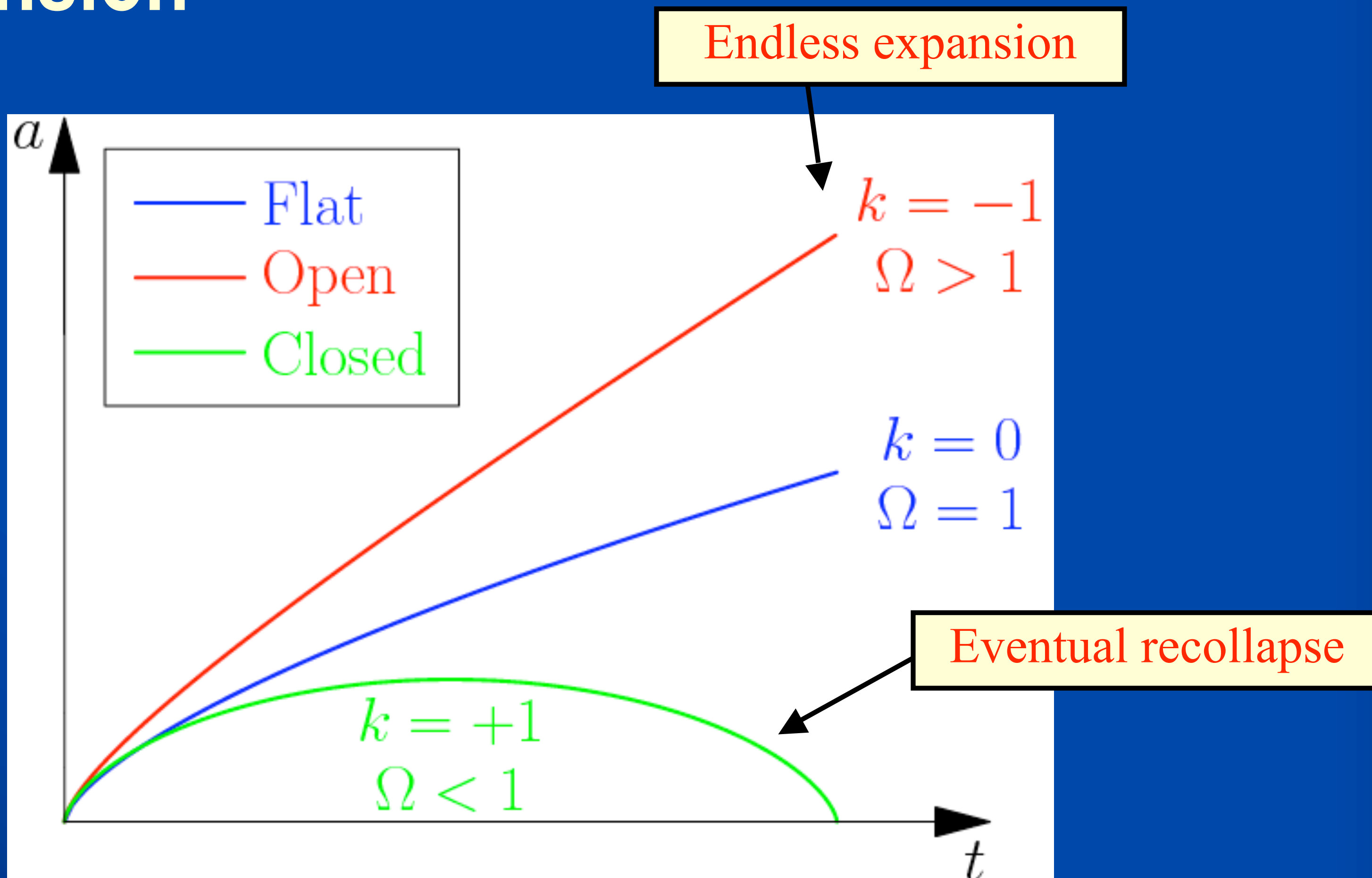
Open ($k > 0$):
- negatively curved
- $\Omega < 1$

Flat ($k = 0$):
- no curvature
- $\Omega = 1$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

$$\Omega \equiv \frac{\rho}{\rho_c}$$

Expansion

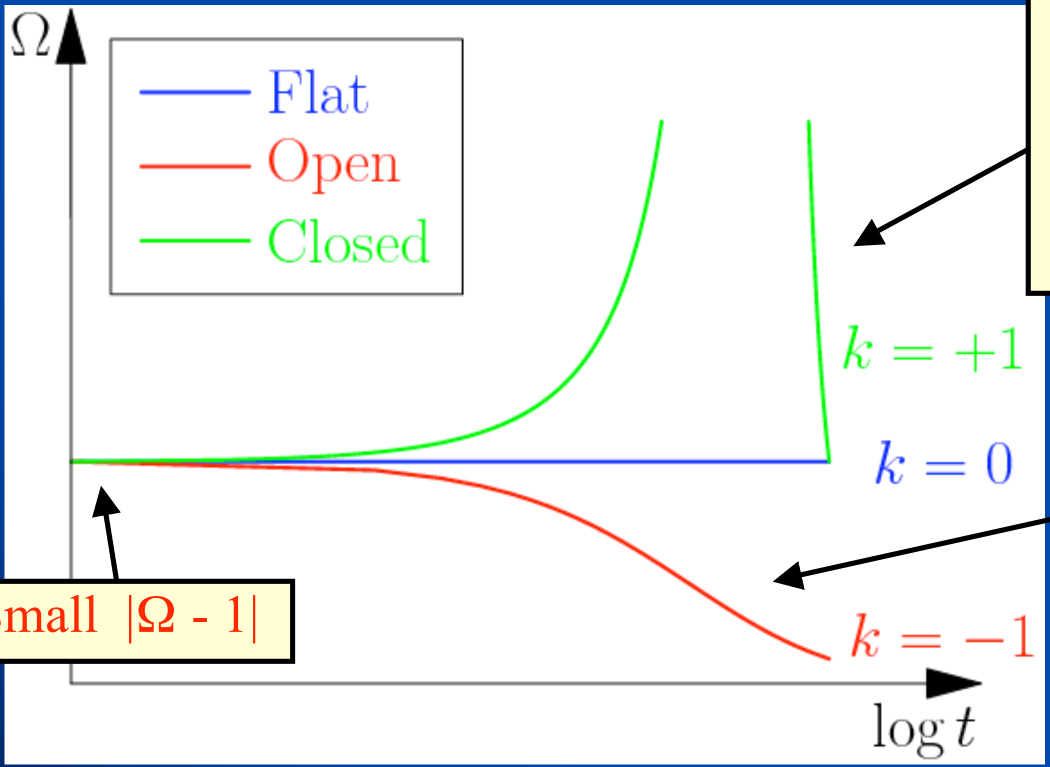


(in presence of matter and radiation only)

The Flatness (Oldness Problem)

$$\Omega(t) = \frac{1}{1 - x(t)}$$

$$x(t) \equiv \frac{k/R^2}{\frac{8\pi G}{3}\rho} \propto \begin{cases} R^2(t) & \text{radiation} \\ R(t) & \text{matter} \end{cases}$$



Closed ($k > 0$):
Initial value of $|\Omega - 1|$ must be small or else universe would have recollapsed already

Open ($k < 0$):
Initial value of $|\Omega - 1|$ must be small or else Ω would now be tiny

($\Omega \sim 1/a$ at late times)

Small $|\Omega - 1|$

3) The Monopole Problem

`t Hooft (1974)
Polyakov (1974)

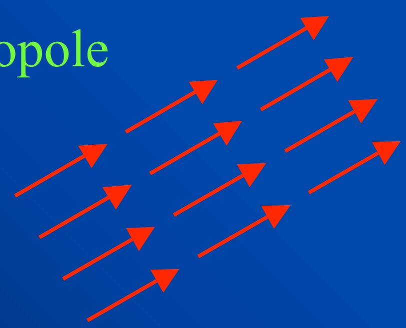
- Imbed $U(1)_{EM} \subset G$. Then, when:

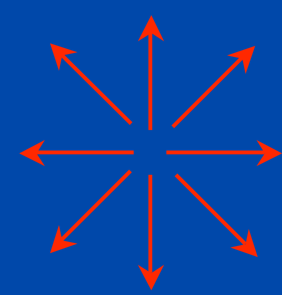
$$G \xrightarrow[\mu]{\text{SSB}} G' \times U(1)$$

there exist gauge/Higgs field configurations topologically stable monopoles of mass $M \sim {}^{\mu}/_{\alpha}$

$$e.g.\ SU(5) \xrightarrow[M \sim 10^{17}\ \text{GeV}]{10^{16}\ \text{GeV}} SU(3)_C \times SU(2)_L \times U(1)_Y$$

No monopole





Magnetic monopole

$B \sim 1/r^2$

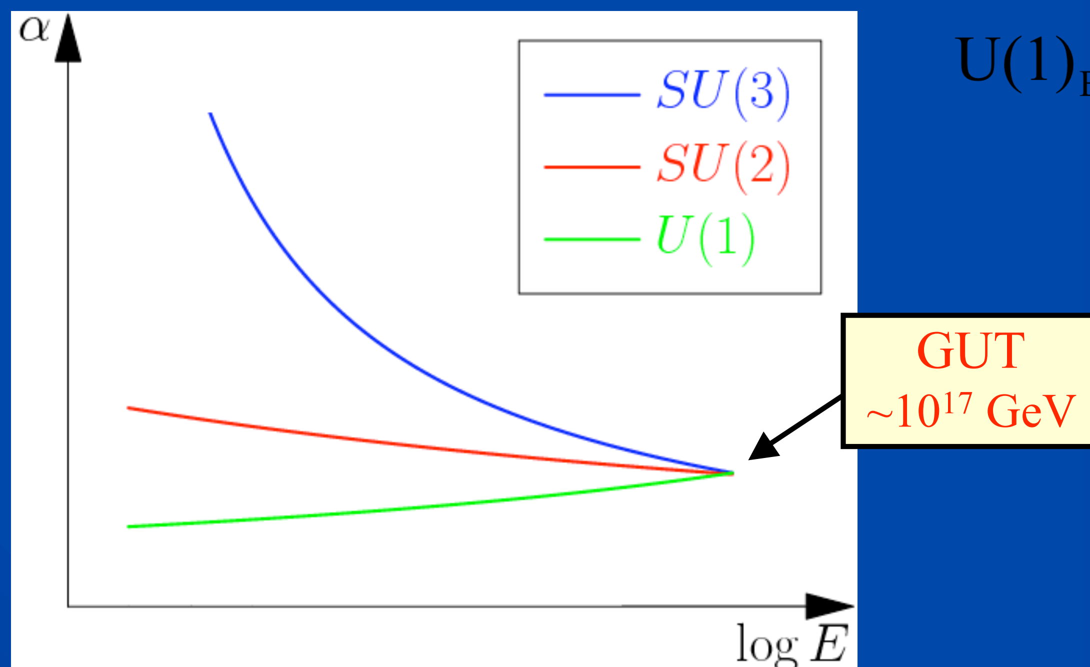
Grand Unified Theory (GUT)

$$\text{GUT} \xrightarrow[\sim 10^{17}\ \text{GeV}]{\text{SSB}} \overset{\text{strong}}{\text{SU(3)}_C} \times \underbrace{\overset{\text{electroweak}}{\text{SU(2)}_L \times \text{U(1)}_Y}}_{\big\downarrow\ \text{SSB}}$$

$$\text{U(1)}_{\text{EM}}$$

Running of the gauge couplings

Unify at high energies (temperatures)



GUT Magnetic Monopoles

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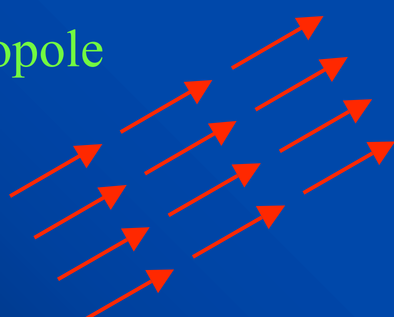
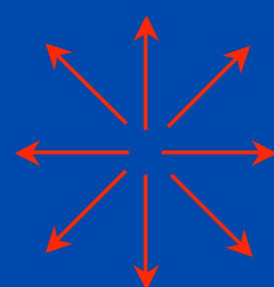
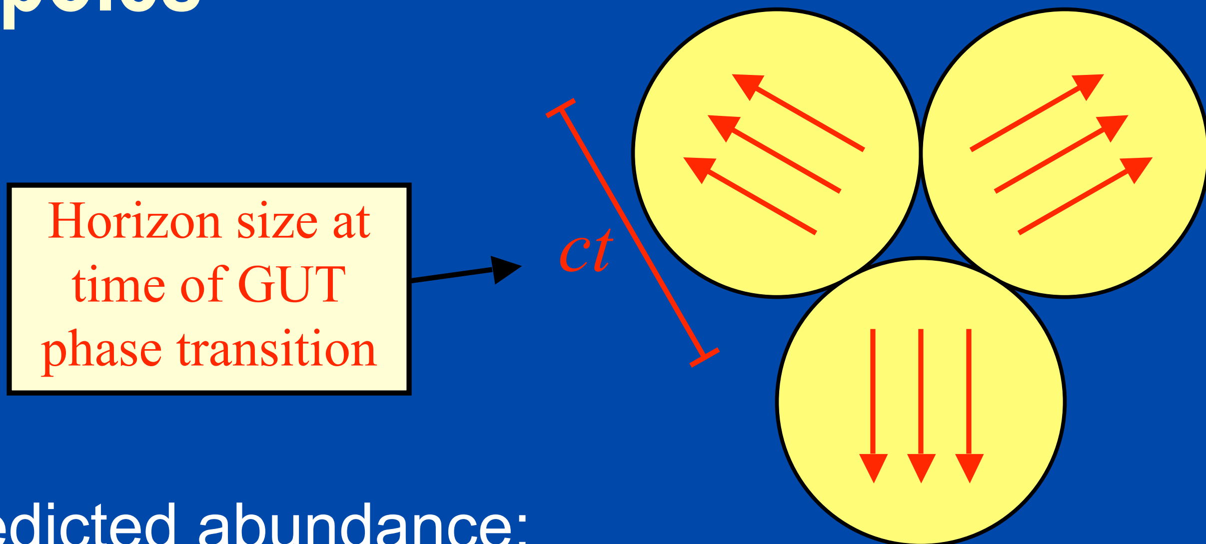




Magnetic monopole

$B \sim 1/r^2$

Monopoles



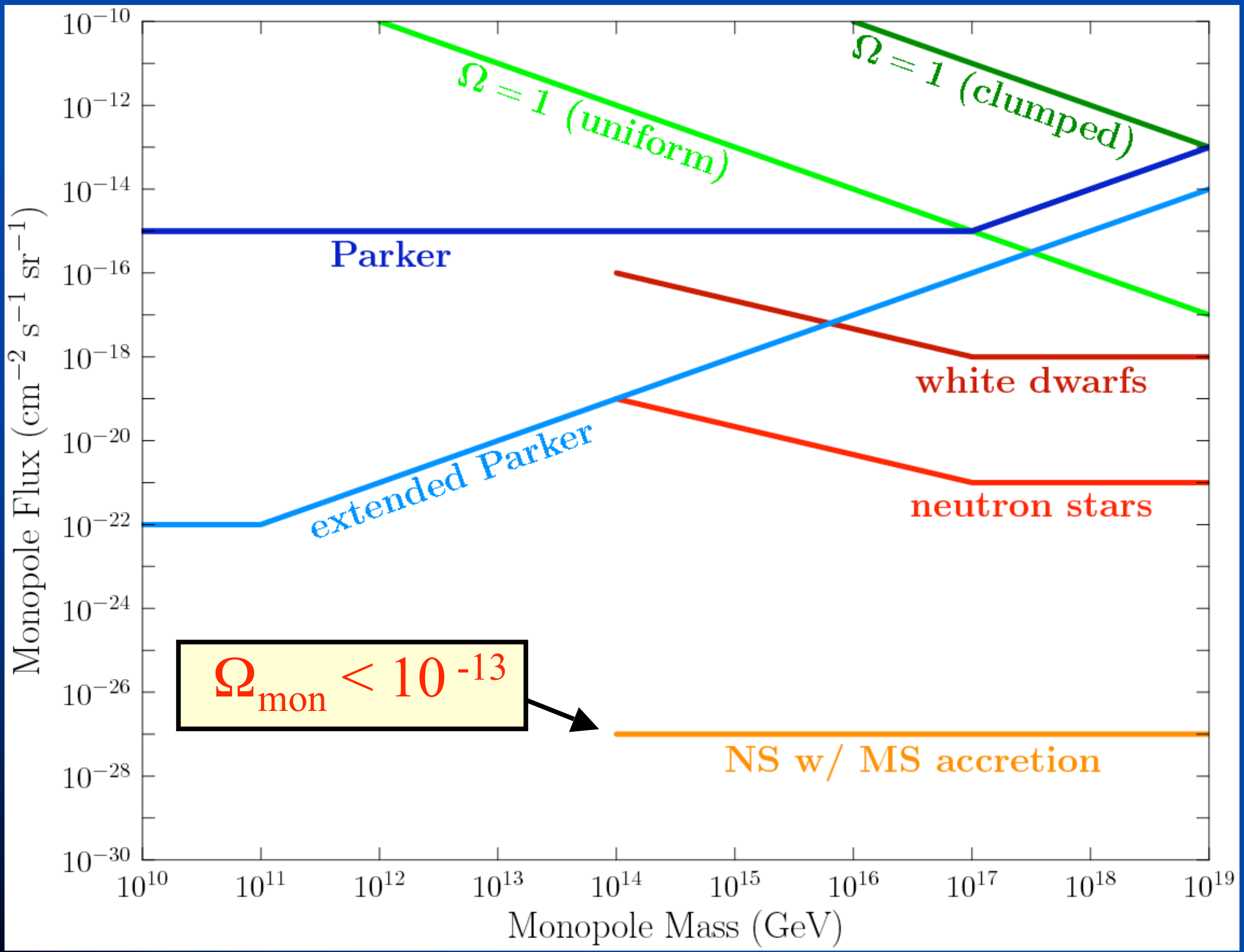
- Predicted abundance:
  ~ 1 / horizon volume (Kibble mechanism)

$\Rightarrow \Omega_{mon} \approx 10^{12}$ NOT OUR UNIVERSE!

Would imply t = 30 yr at T = 3K

Monopoles

Astrophysical bounds:
Far more severe than direct searches



Parker bound (due to survival of galactic magnetic field): Parker (1970); Turner, Parker & Bogdan (1982)

WD:
Freese (1984)

NS:
Kolb, Colgate & Harvey (1982); Dimopoulos, Preskill & Wilczek (1982); Freese, Turner & Schramm (1983)

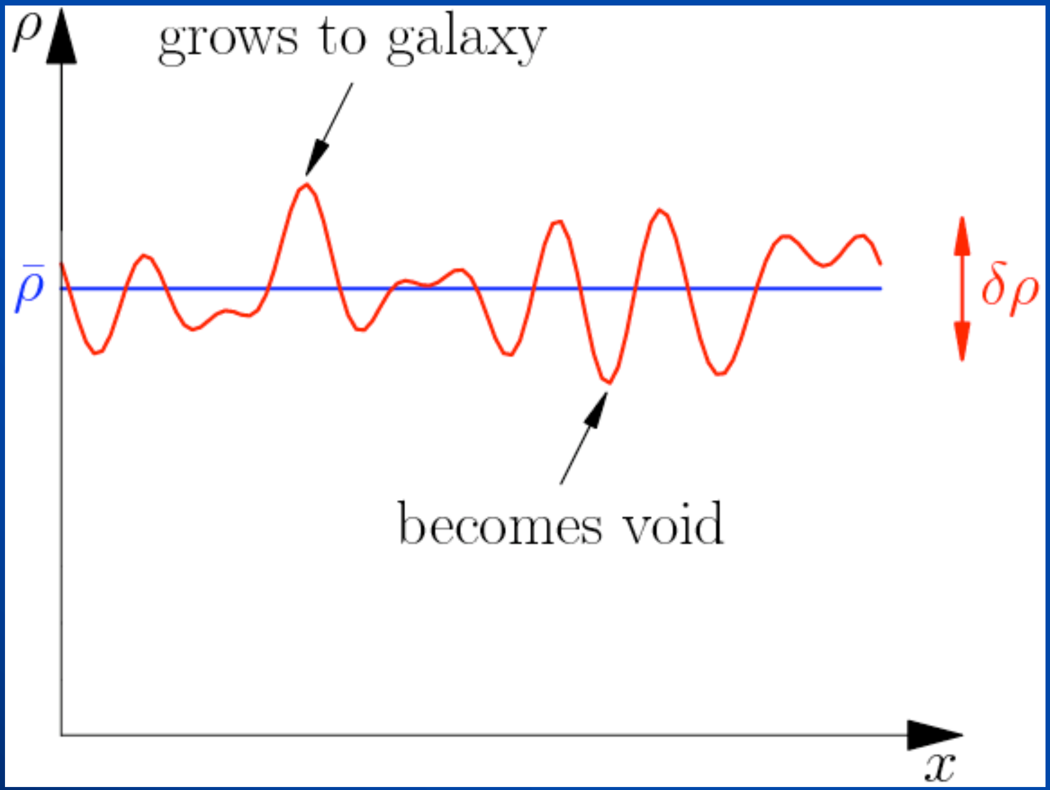
NS w/ MS accretion:
Frieman, Freese & Turner (1988)

Extended Parker:
Adams, Fatuzzo, Freese, Tarle, Watkins & Turner (1993); Lewis, Freese & Tarle (2000); [Particle Data Book]

1a) Inhomogeneities: galaxies, clusters, large scale structure

- Small density fluctuations produced in inflation can give rise to structures that we see.

$\delta\rho / \rho$



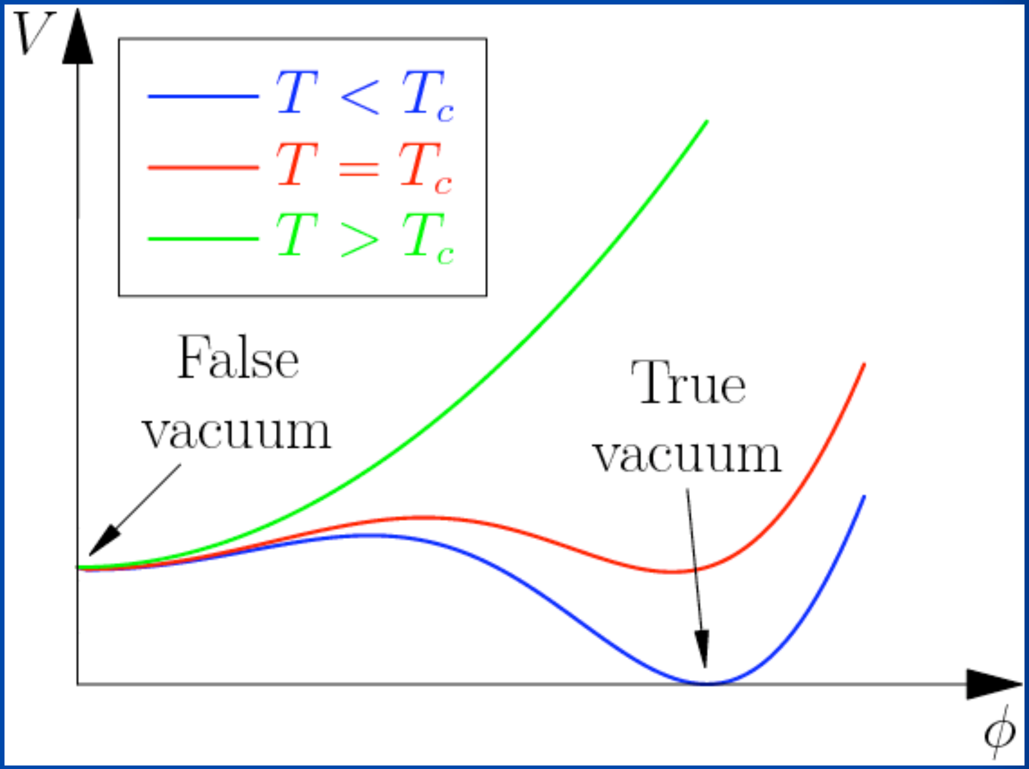
Cosmological Problems addressed by inflation:

- 1) Homogeneity and isotropy of the universe
- 2) Flatness/oldness of universe
- 3) Excess magnetic monopoles produced at Grand Unified phase transition
- 1a) Inhomogeneity: origin of density fluctuations that give rise to large scale structure

The Solution: Inflation
Original Proposal:Old Inflation

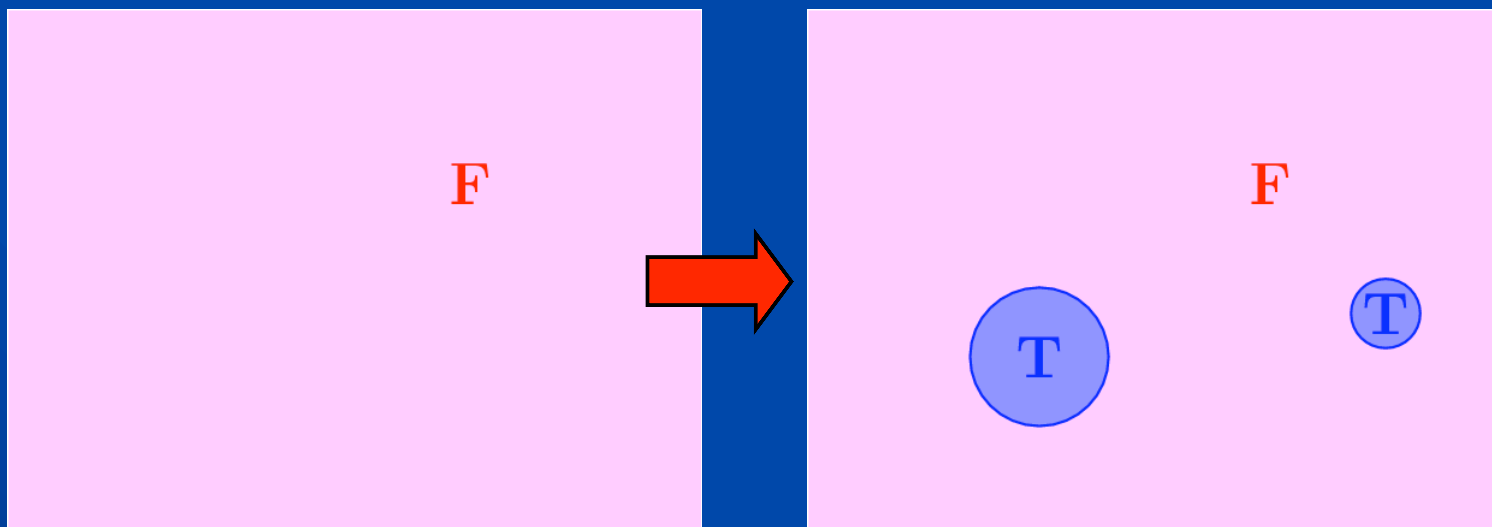
Guth (1981)

- Temperature-dependent potential
- Initially at global minimum $\phi = 0$
- When $T < T_c$, no longer global minimum $\Rightarrow$ false vacuum



Old Inflation

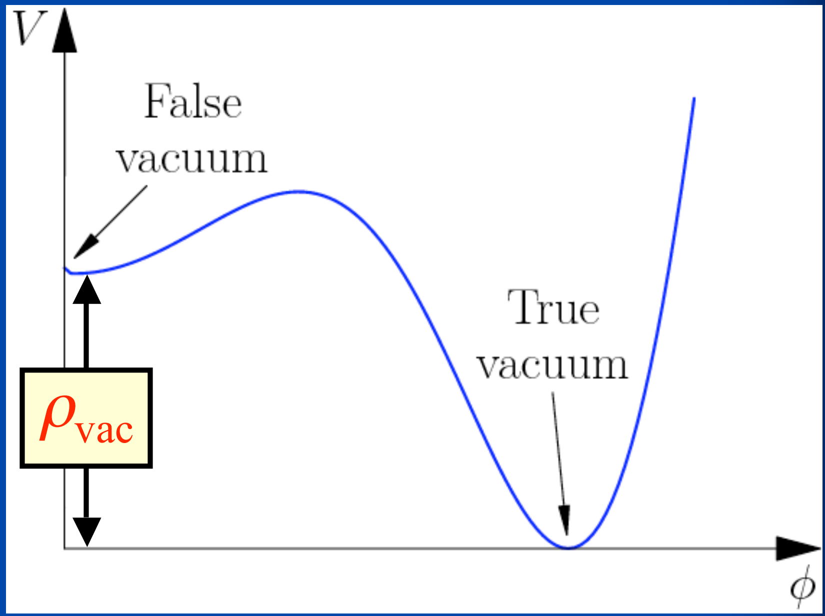
- Vacuum decay



Entire universe is in false vacuum (**F**)

Nucleate bubbles of true vacuum (**T**)

Old Inflation



- With tunneling, the nucleation rate is slow, so the universe is trapped in the false vacuum
  for a long time

- The vacuum energy dominates over matter and radiation
  ⇒ de Sitter-like expanding universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{vac}} = \text{constant}$$

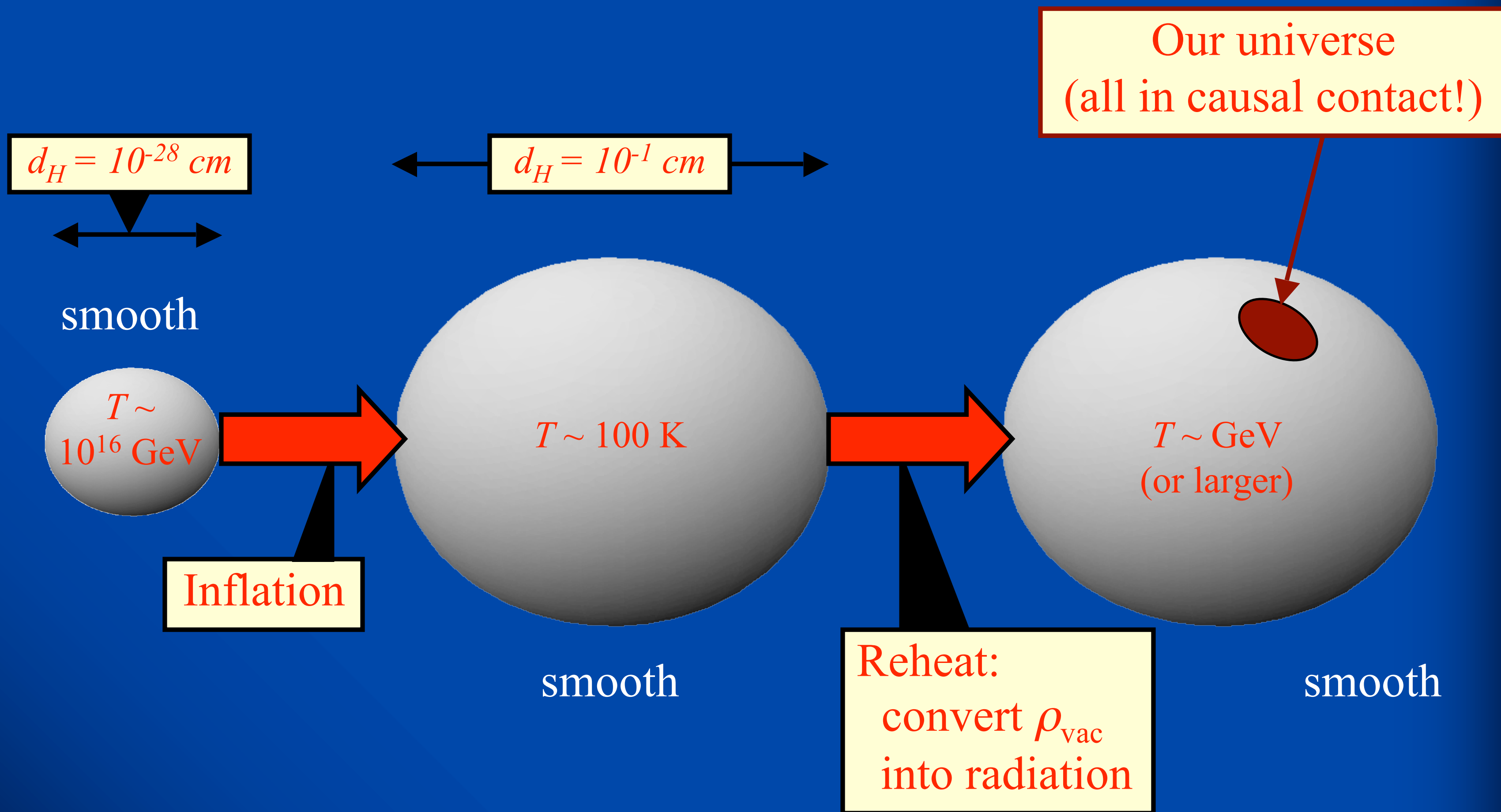
$$\text{solution}: \quad a \sim e^{Ht}, \quad H = \left[\frac{8\pi G}{3}\rho_{\text{vac}}\right]^{1/2}$$

- Enough inflation to solve problems:

$$a_{\text{end}} = a_{\text{begin}} \times 10^{27} = a_{\text{begin}} \times e^{65}$$

Horizon Volume

$$d_H = ct$$



Inflation Parameters

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{vac} - \frac{k}{R^2}$$

Before	During	After
R_0	$R_0\, e^{Ht}$	$e^{65}\, R_0 = 10^{27}\, R_0$
$T_0 \sim 10^{16}$ GeV	$T_0\, e^{-Ht}$	100 K (reheat) → ≥ GeV
k / R_0^2	$(k / R_0^2)\, e^{-Ht}$	$10^{-54}\ (k / R_0^2)$
$x \equiv \dfrac{k / R^2}{\frac{8\pi G}{3}\rho_{vac}}$		$10^{-54}\, x$
$\Omega \equiv 1/(1-x)$		$\Omega = 1.00000\ldots$
$d_H \equiv ct \sim 10^{-28}$ cm		~ 0.1 cm

Inflation Resolves Cosmological Problems

- Horizon Problem (homogeneity and isotropy): small causally connected region inflates to large region containing our universe
- Flatness Problem $k/a^2 \rightarrow \mathrm{small}$ $\Omega \rightarrow 1$
- Monopole Problem: tightest bounds on GUT monopoles from neutron stars (Freese, Schramm, and Turner 1983); monopoles inflated away (outside our horizon)
- BONUS: Density Perturbations that give rise to large scale structure are generated by inflation

Theoretical Models of Inflation

- ***1) Tunneling Models:***

  Why Old Inflation Failed

  New proposals for tunneling models:

  (i) double-field inflation (Adams and Freese)

  (ii) extended inflation (Steinhardt)

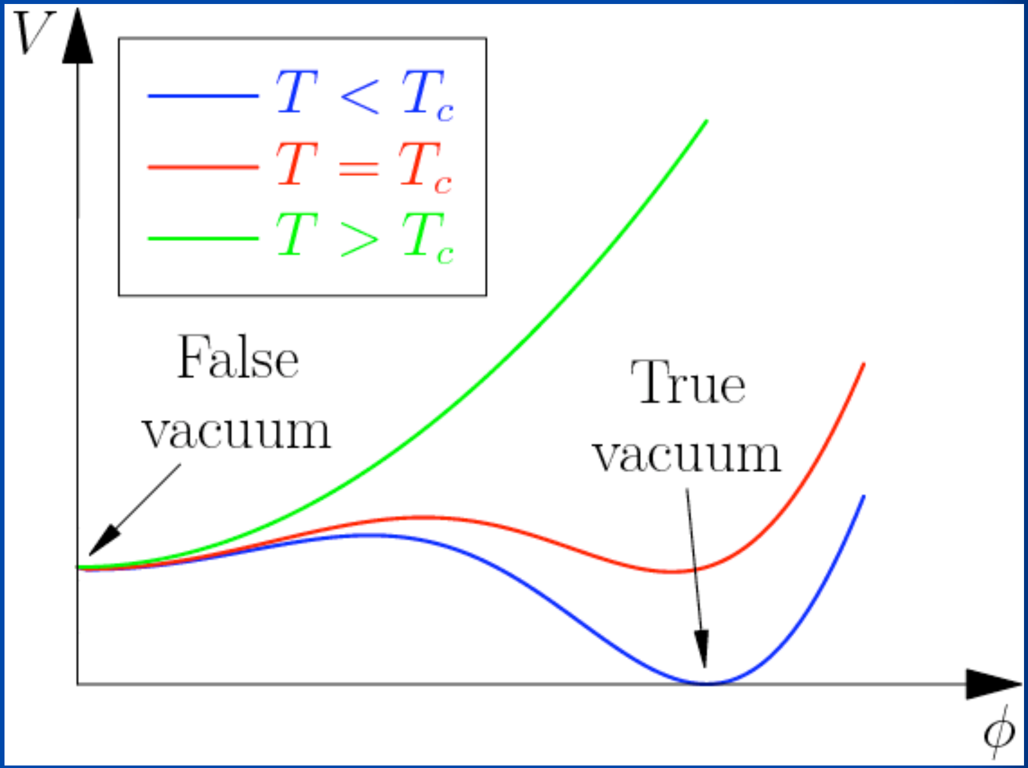
  (iii) Chain inflation (Freese and Spolyar)
- ***2) Rolling Models:***

  new inflation, chaotic inflation, hybrid inflation
- Natural inflation (Freese, Frieman, Olinto)

Old Inflation

Guth (1981)

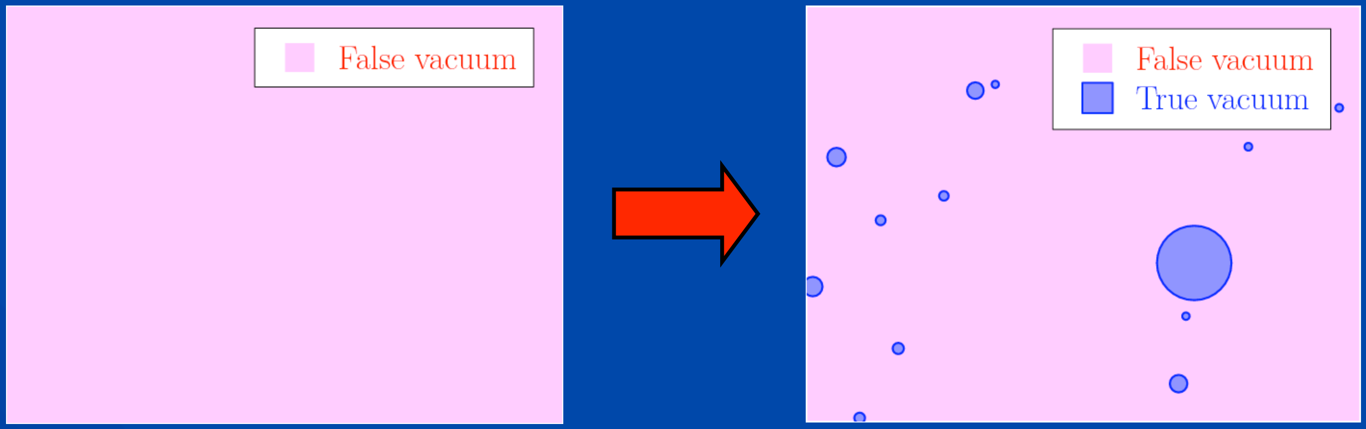
- Universe goes from false vacuum to true vacuum.



- Bubbles of true vacuum nucleate in a universe of false vacuum
  (first order phase transition)

Old Inflation

- Vacuum decay: "swiss cheese problem"



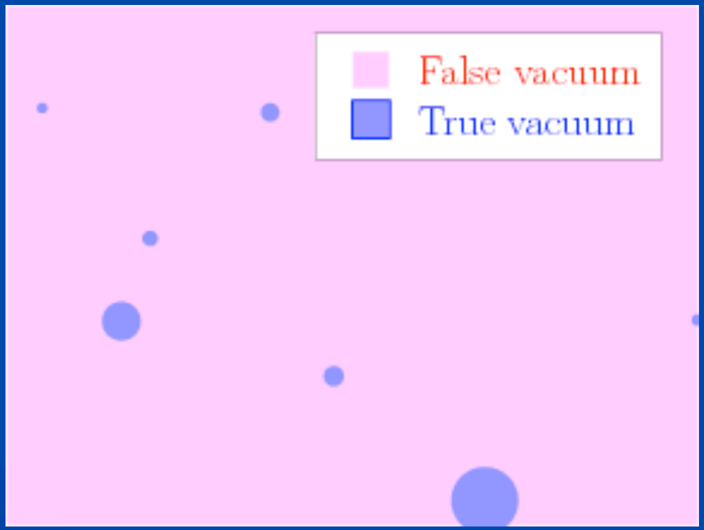
Entire universe is in false vacuum

Nucleate bubbles of true vacuum

Problem: bubbles never percolate & thermalize
$\Longrightarrow$ NO REHEATING

Old Inflation

- Bubbles inflate away faster than they form & grow
  $\Rightarrow$ no end to inflation & no reheating

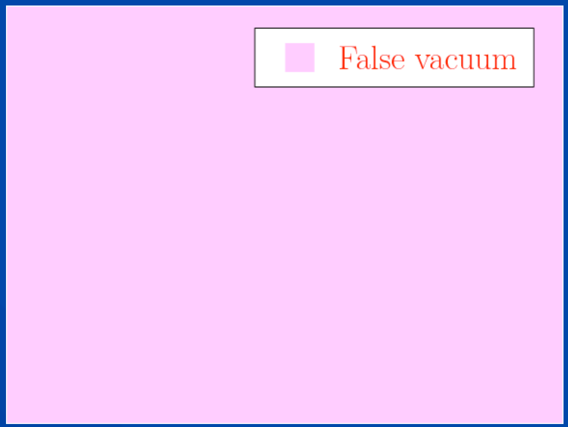


What is needed for tunneling inflation to work?

- Two requirements for inflation:
- 1) Sufficient Inflation: 60 e-foldings
- 2) The universe must thermalize and reheat; i.e. the entire universe must go through the phase transition at once. Then the phase transition completes.
- Can achieve both requirements with
- (i) time-dependent nucleation rate in Double-field inflation (Adams and Freese '91) with two coupled fields in a single tunneling event
- (ii) Chain Inflation (Freese and Spolyar 2005) with multiple tunneling events

Rapid phase transition leads to percolation (entire universe goes through phase transition at once)

- Vacuum decay: "swiss cheese problem"



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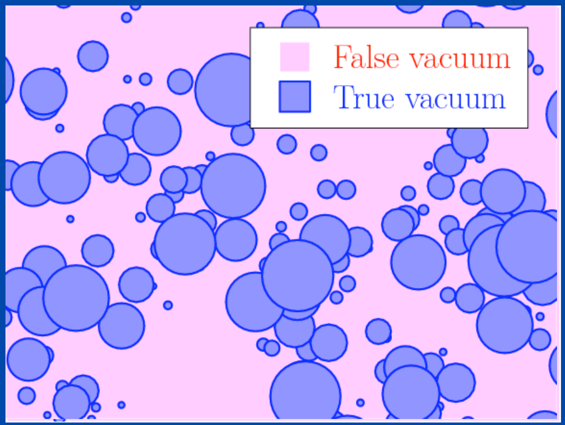
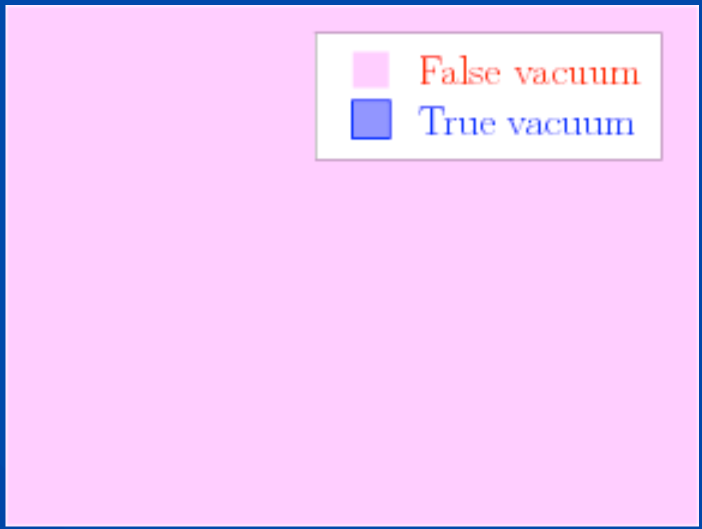


Nucleate bubbles of true vacuum

Rapid Phase Transition

- Need bubbles to form and grow faster than inflation $\Rightarrow$ inflation comes to an end and reheating occurs



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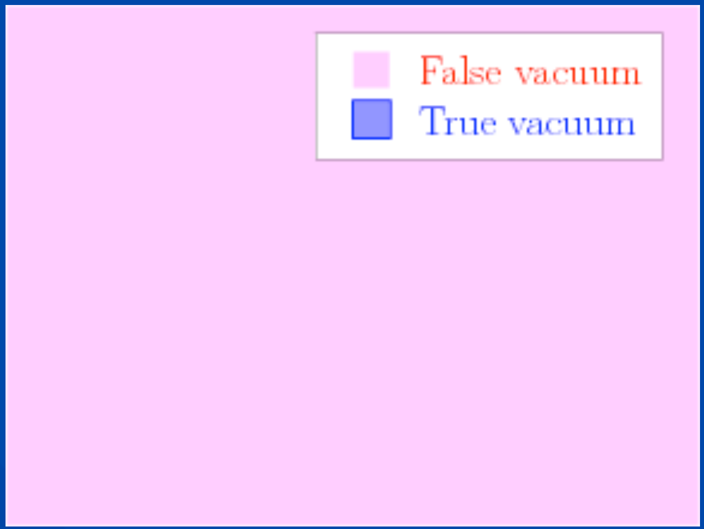
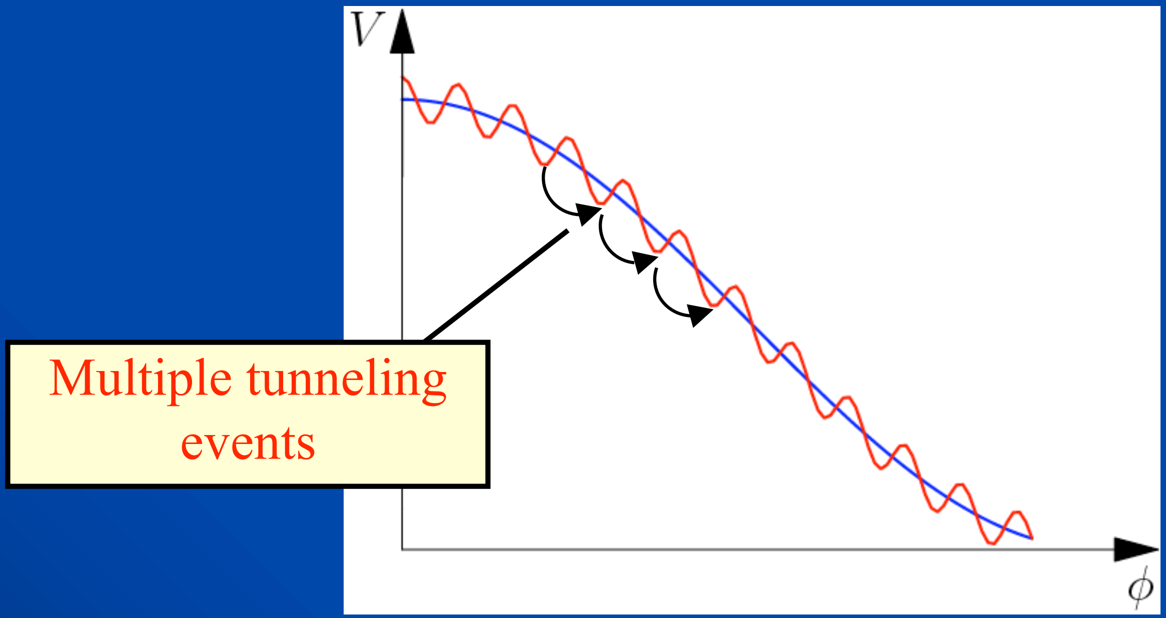


Chain Inflation

Freese & Spolyar (2005)



Relevant to:

- stringy landscape
- QCD (or other) axion

- Graceful exit:
  requires that the number of e-foldings per stage is $N < 1/3$
- Sufficient inflation:
  total number of e-foldings is $N_{tot} > 60$

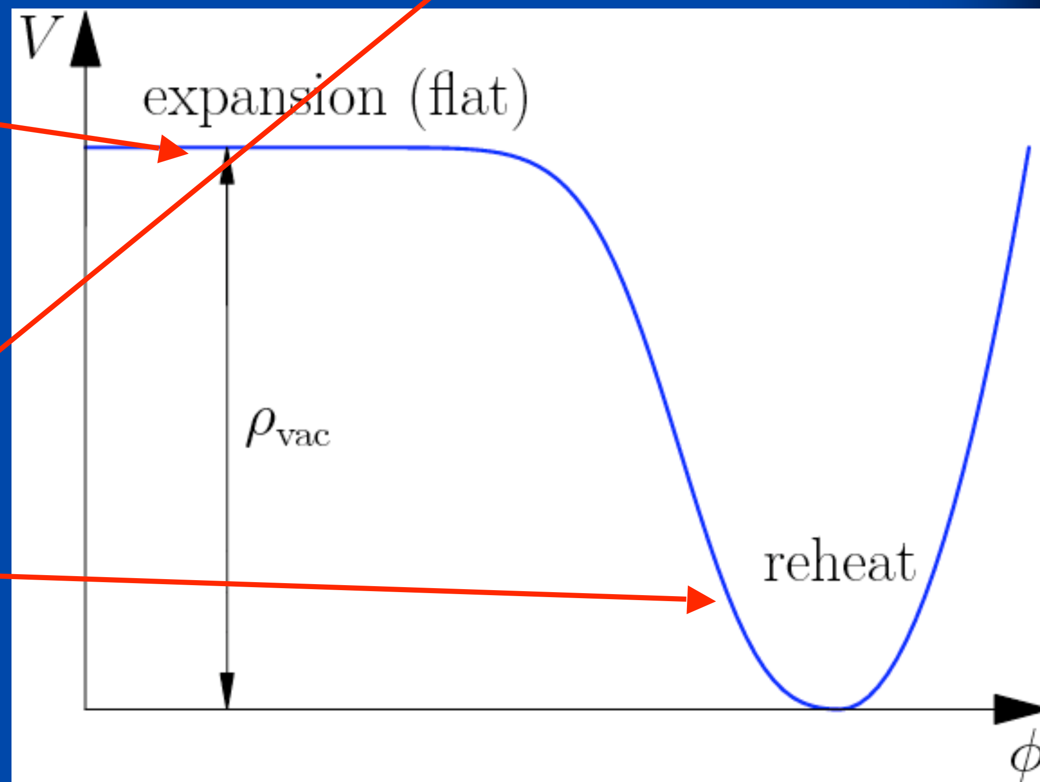
Second Class of Models: Rolling Models of Inflation

Linde (1982)
Albrecht & Steinhardt (1982)

- Equation of motion: $$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0$$
- Flat region:
  - V almost constant
  - ρ_{vac} dominates energy density

$$\rightarrow a \approx a_i \, e^{Ht}$$

- Decay of ϕ:
  - Particle production
  - Reheating



Examples of Rolling Models

- New Inflation (Linde 1982, Albrecht and Steinhardt 1982)
- Chaotic Inflation (Linde1983)
- Power Law Inflation (Lucchin and Mattarese 1985, Liddle 1989)
- Natural Inflation (Freese, Frieman, Olinto 1990)

Different models make different predictions for data.

Inflation is a very nice theoretical framework.

Now it is time to test the model.

On: the role of observations

“Faith is a fine invention
When Gentlemen can see ---
But Microscopes are prudent
In an Emergency

Emily Dickinson, 1860

From Theory to Observation: Predictions of Inflation

- 1) flat universe: $\Omega = 1$
- 2) Specrum of density perturbations:

$$|\delta_k|^2 \propto k^n, \ n \sim 1$$

- 3) gravitational wave modes
- Individual models make specific predictions.
- Can test inflation as a concept and can differentiate between models.

Prediction 1 of Inflation:

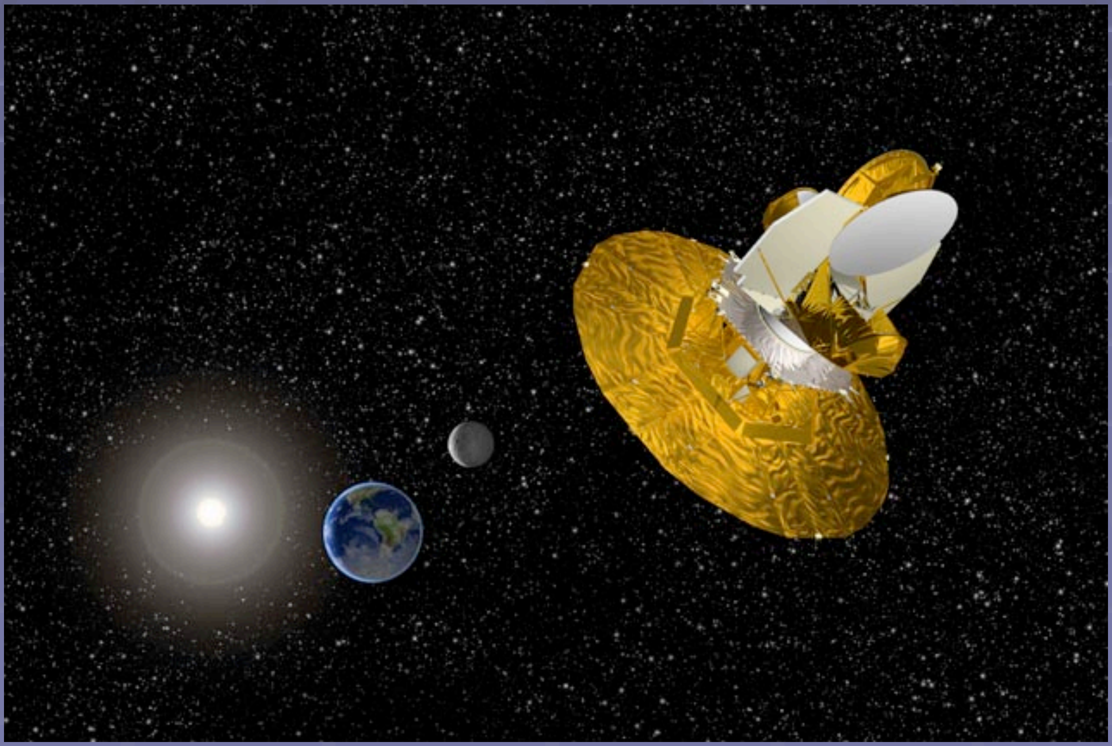
The geometry of the universe is flat;

i.e. the density is critical and

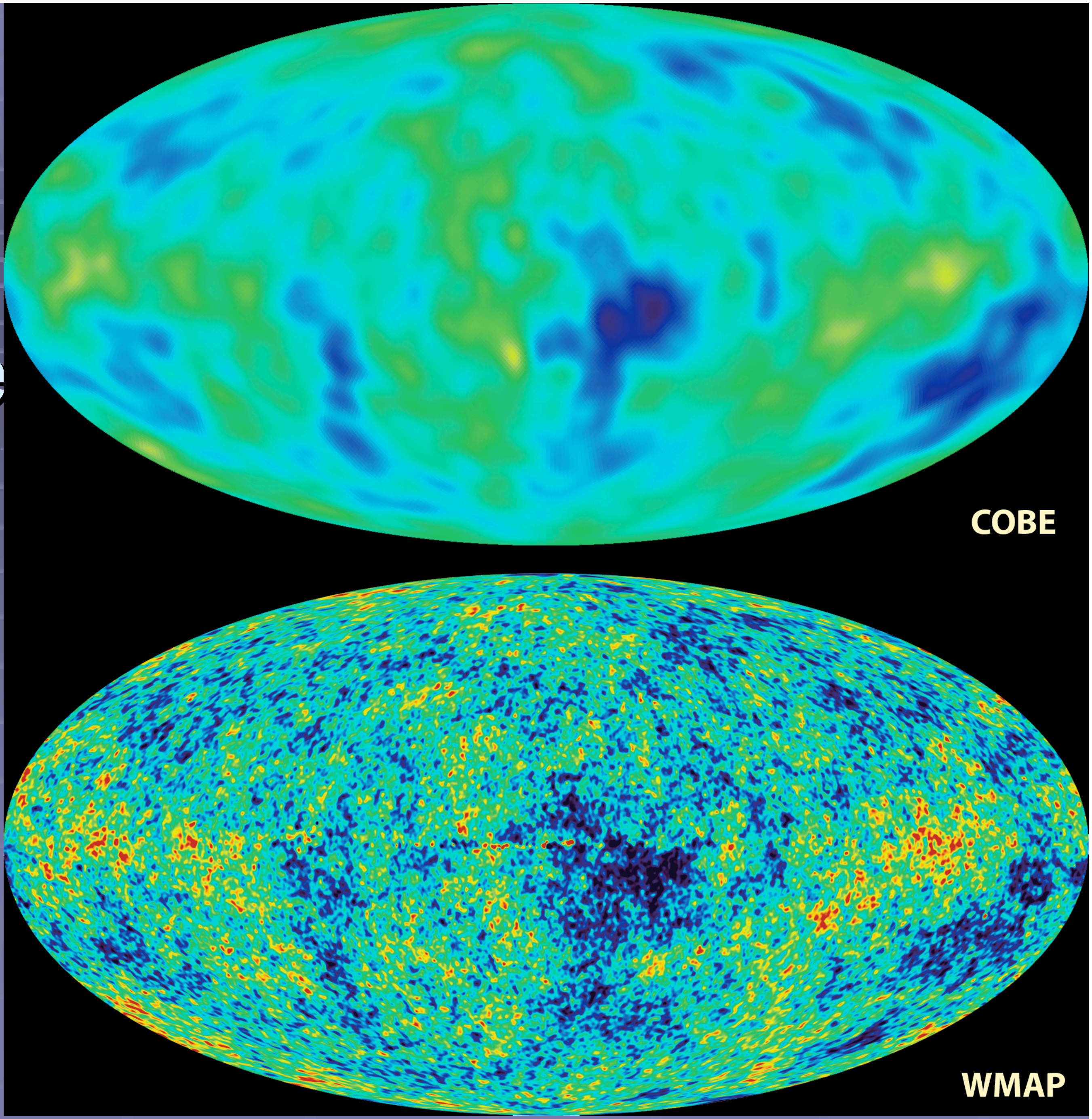
$$\Omega = 1$$

WMAP Satellite

- Launched June 2002
- Data released Feb. 2003



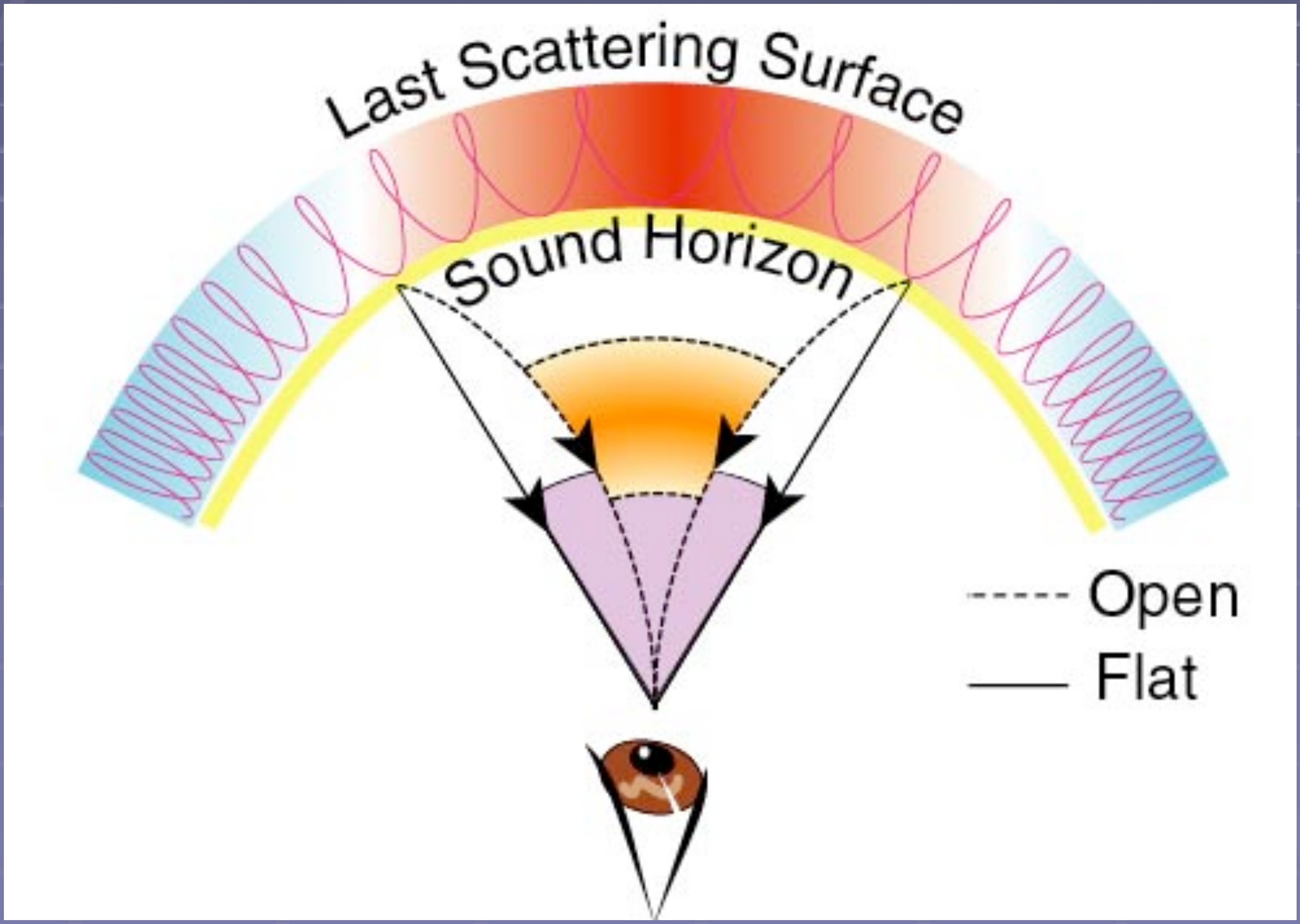
The Microwave Sky



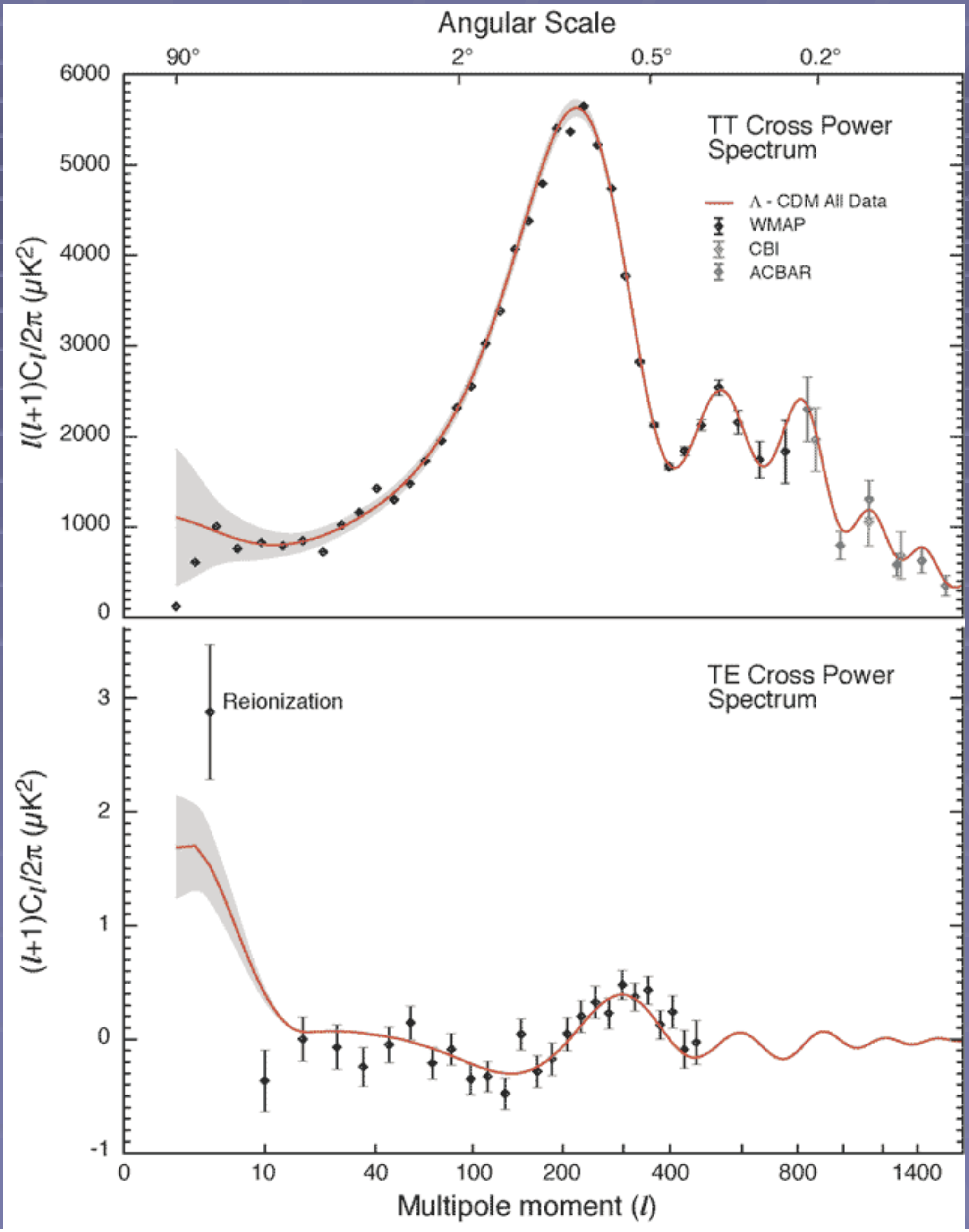
The Doppler Peak

- Acoustic oscillations in the photon/atom fluid are imprinted at last scattering. We expect a peak in the microwave background at the sound horizon (distance sound could travel in the age of the universe).
- If the universe is flat, the peak is at one degree.
- If the universe is a saddle, the peak is at less than one degree.

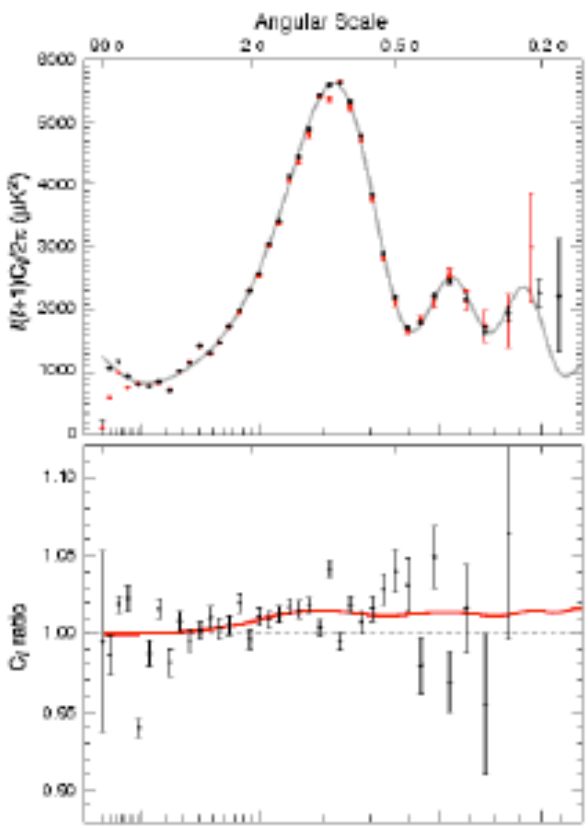
How can Microwave Background tell us about geometry?



Doppler Peak at 1 degree (WMAP1)



Comparison to First-year Spectrum



top:

Black: 3-year; red: 1-year

bottom:

ratio: 3-year/1-year

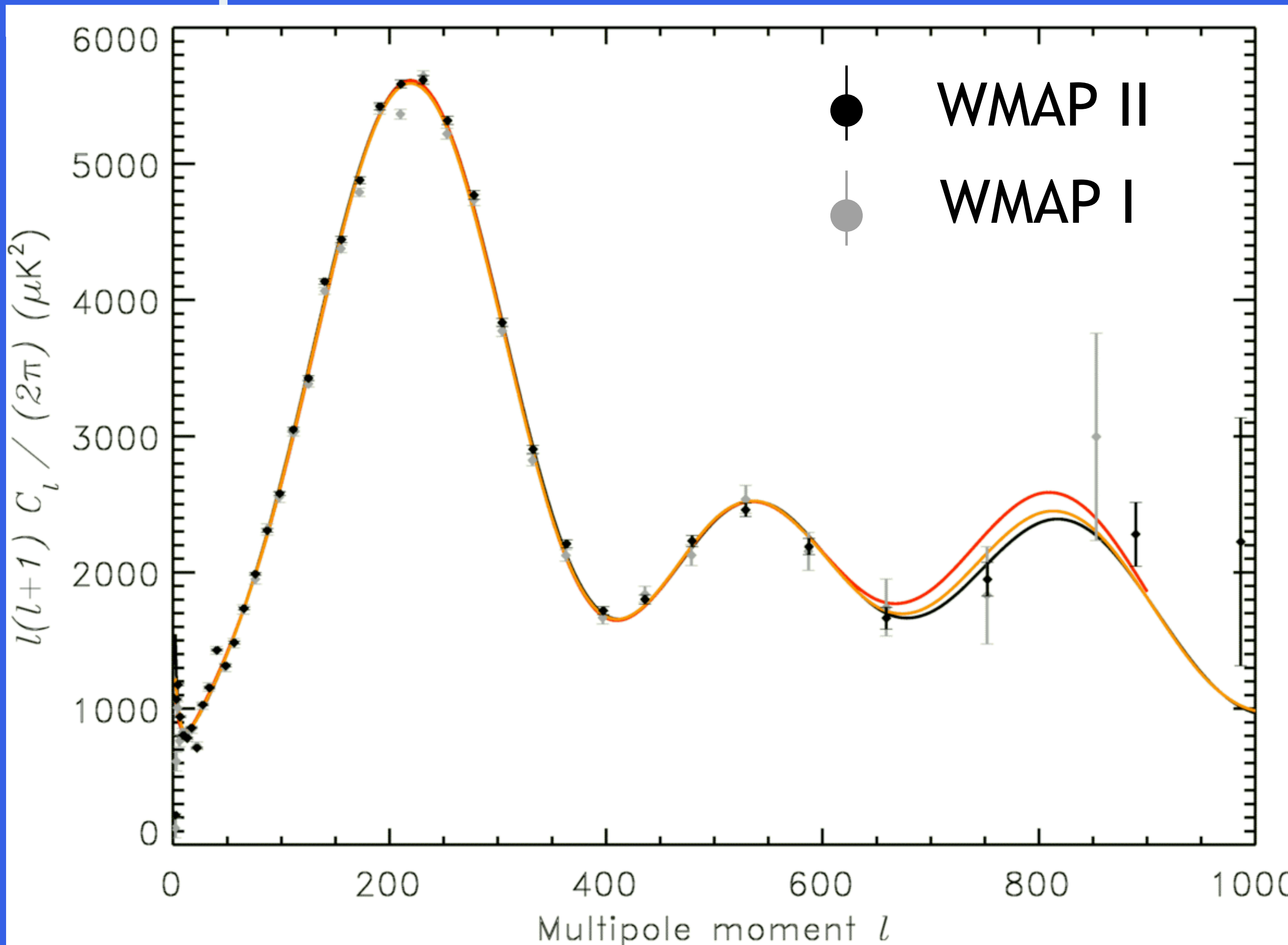
high l: noise reduced by a factor of >3. (3 times more data and finer sky map pixels)

intermediate l: improvement in modeling beam response raises spectrum 1-2%

low l: improvement in power estimation from maps with sky cuts (l<10). l=2 still low, l=3 changes by factor of ~2.

Ratio at left shows low-l results with fixed methodology - only 2-3% change in sky map data.

Comparison with WMAP I



3 year:
Black curve.
Longer
integration
times,
smaller
Pixels

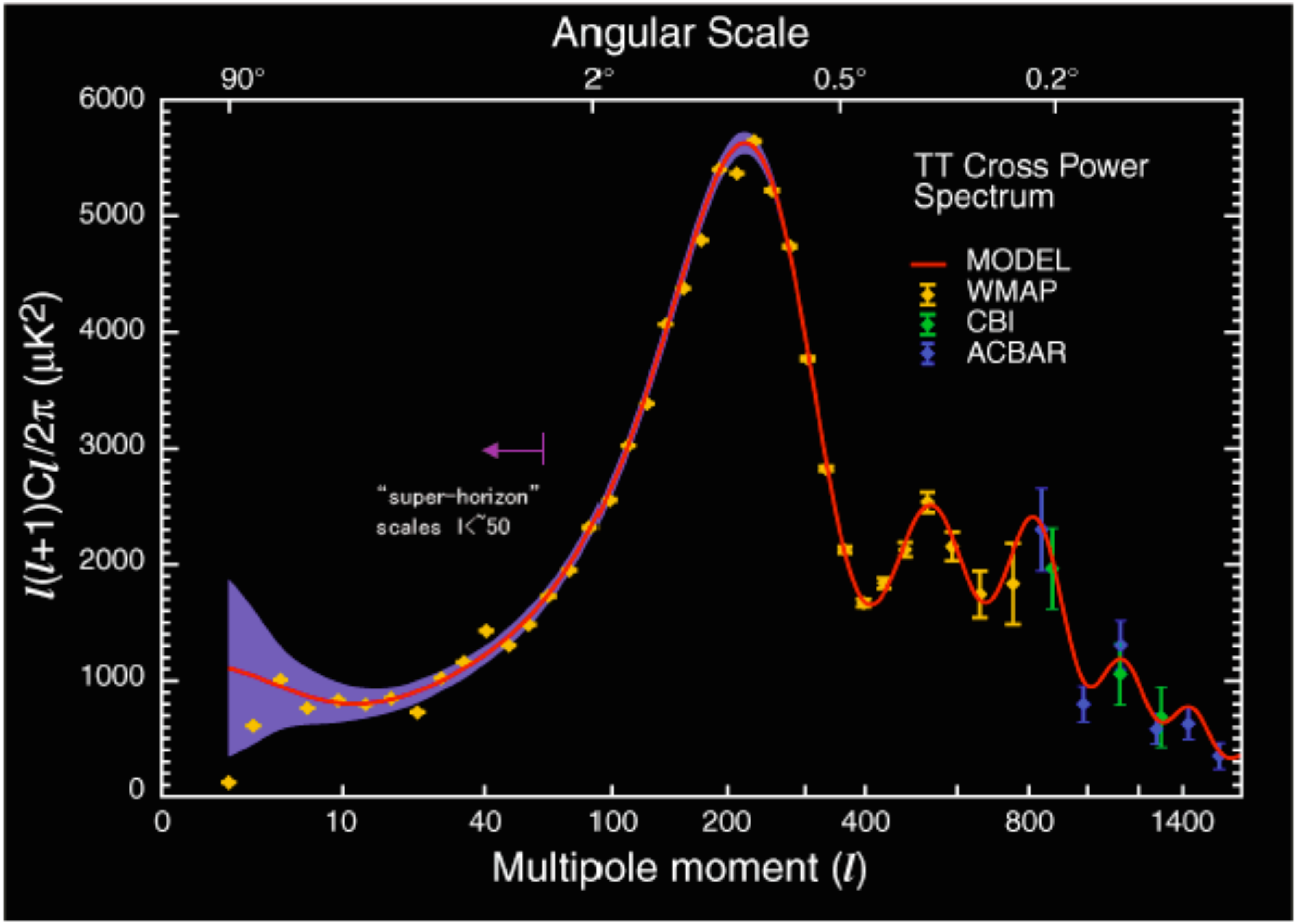
n.b. MOND
is a very
poor fit
to large-l
third peak

Best fit LCDM WMAP I

Best fit LCDM WMAP I Ext

Best fit LCDM WMAP II

PREDICTION OF INFLATION: SUPERHORIZON MODES

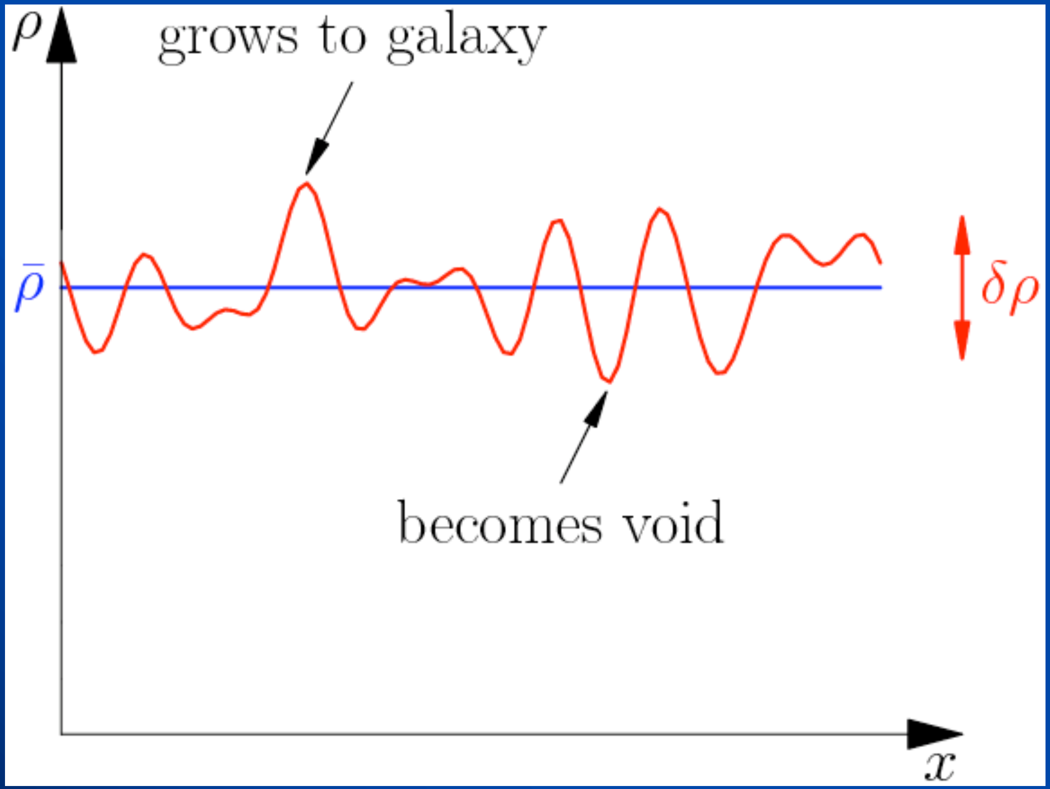


Prediction 1 is confirmed

- WMAP confirms the inflationary prediction that $\Omega = 1$

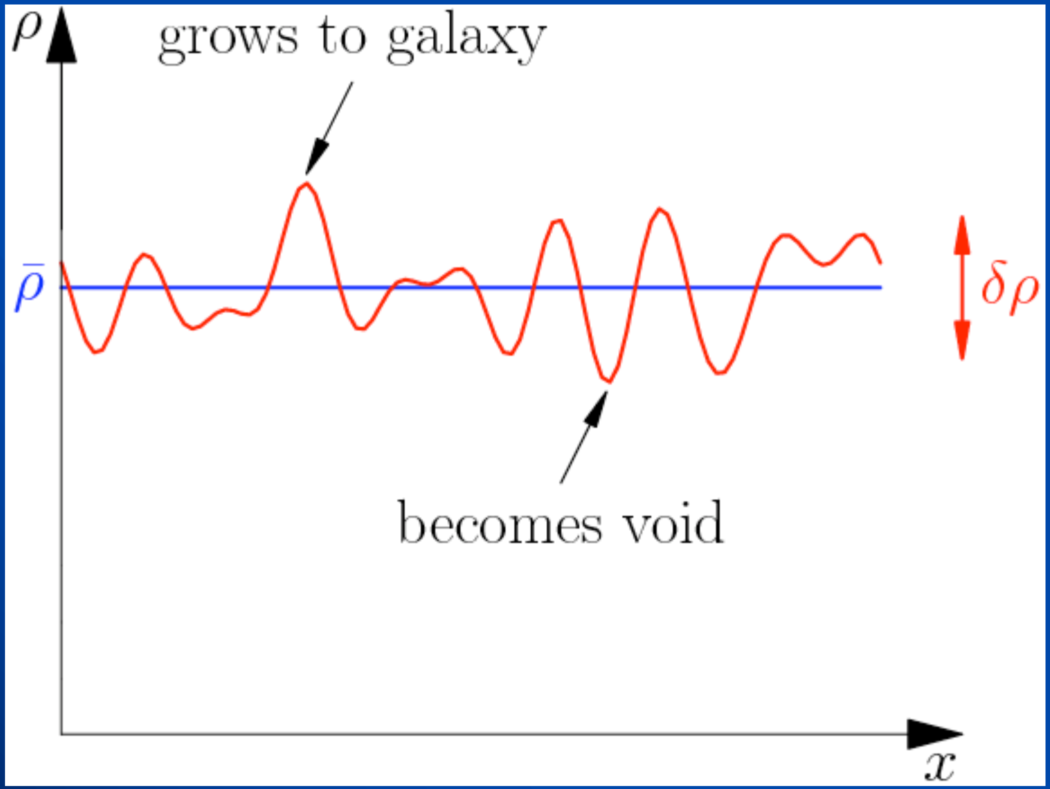
Prediction 2 of Inflation: Density Fluctuations

- Density fluctuations are produced in rolling models of inflation $\delta\rho / \rho$
  - Origin: quantum fluctuations $\left\langle \Delta\phi^2 \right\rangle \sim H / 2\pi$



Density Fluctuations

- Different regions of the universe start at different values of ϕ, take different times to reach bottom
  $\Rightarrow$ end at different energy densities



Density Perturbations

Hubble radius

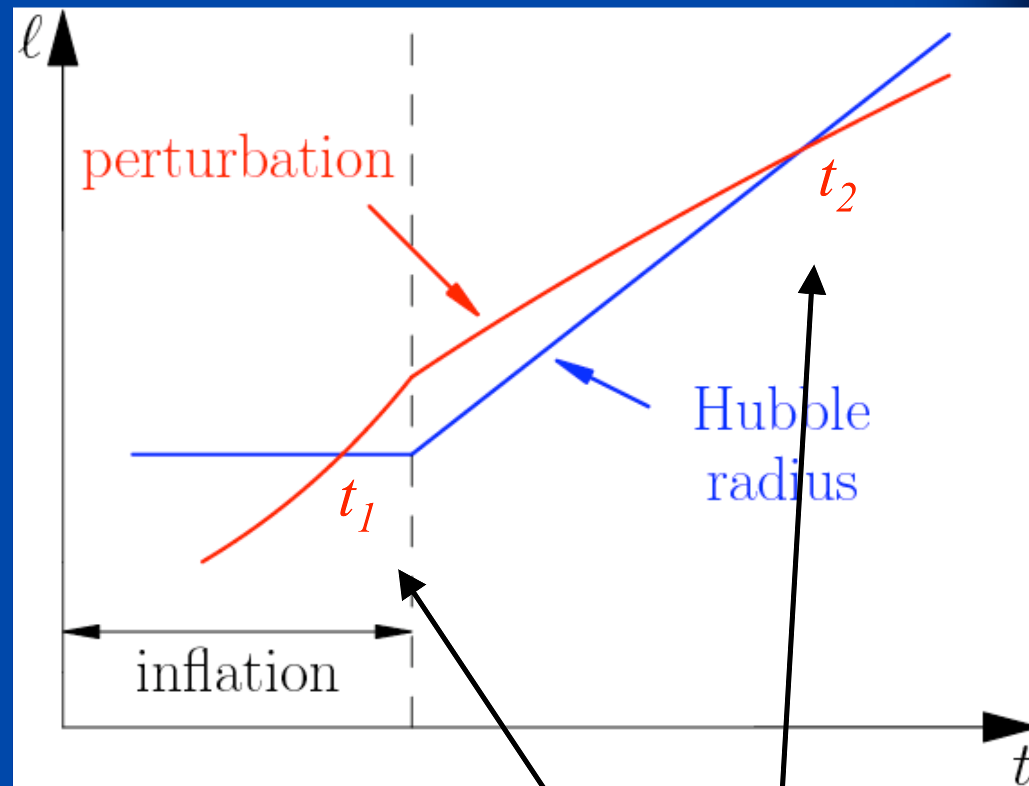
$$\ell_H \sim \frac{1}{H} \sim \begin{cases} \text{constant} & \text{during inflation} \\ t & \text{post inflation} \end{cases}$$

Perturbation

$$\lambda_{\text{pert}} \sim \begin{cases} e^{t} & \text{during inflation} \\ t^{2/3} & \text{post inflation} \end{cases}$$

Two horizon crossings

Causal microphysics before t_1 describes density perturbations at t_2



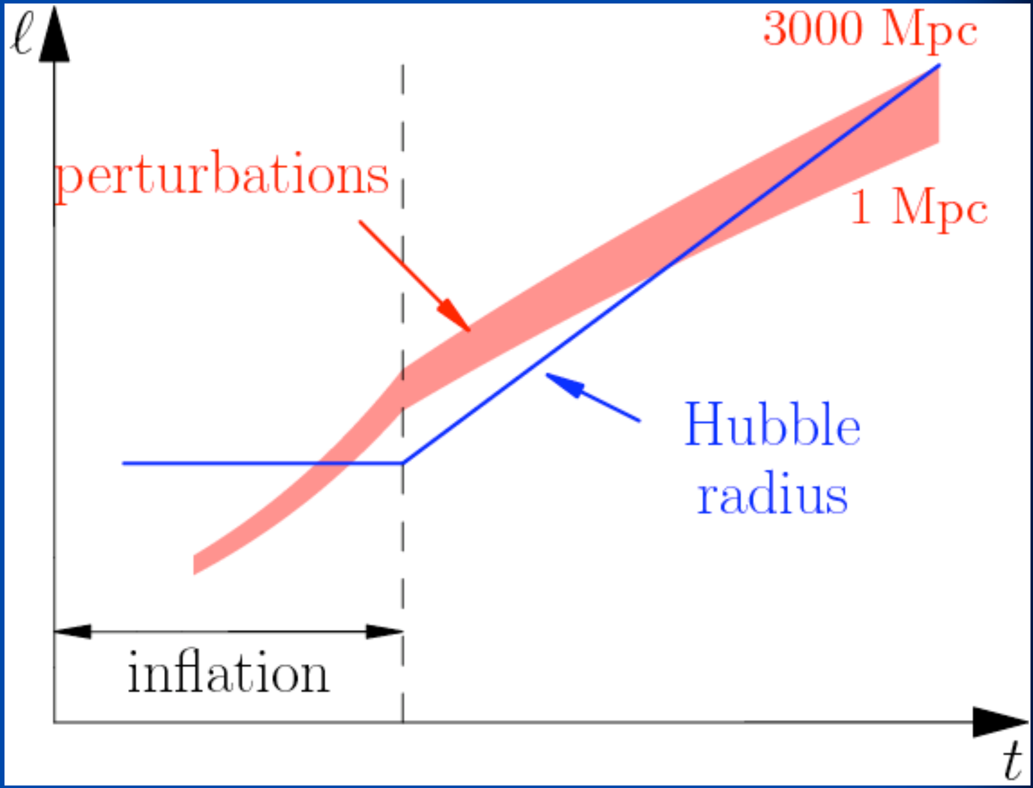
$$\left.\frac{H^2}{\dot{\phi}}\right|_{t_1} = \left.\frac{\delta\rho}{\rho}\right|_{t_2}$$

Density Perturbations

Scales of structure in Universe:

- Distance between galaxies
  ~ 1 Mpc

- Horizon size (size of our observable universe)
  ~ 3000 Mpc



Density Perturbations

Lead to test of inflation theory:

- Must match amplitude of observations

$$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T} \sim 10^{-5}$$

- Must match spectrum of observations (amplitude on all length scales)

Spectrum of Perturbations

- Fourier Transform $$\frac{\delta\rho}{\rho} \xrightarrow[F.T.]{} \delta_k$$
- Power Spectrum $$P_k = |\delta_k|^2 \sim k^n$$
  - n = 1: equal power on all scales (when perturbations enter the horizon)
    Harrison-Zel'dovich-Peebles-Yu
  - n < 1: extra power on large scales

Spectrum of Perturbations

- Power Spectrum $P_k = |\delta_k|^2 \sim k^n$
- During inflation, H and $d\phi/dt$ vary slowly

$$\frac{\delta\rho}{\rho}\text{(when entering horizon)} = \frac{H^2}{\dot{\phi}}\text{(exiting horizon during inflation)}$$

~ same on all scales

- Predicts $n \sim 1$: CORRECT
- Precise predictions of n in different models leads to test of models

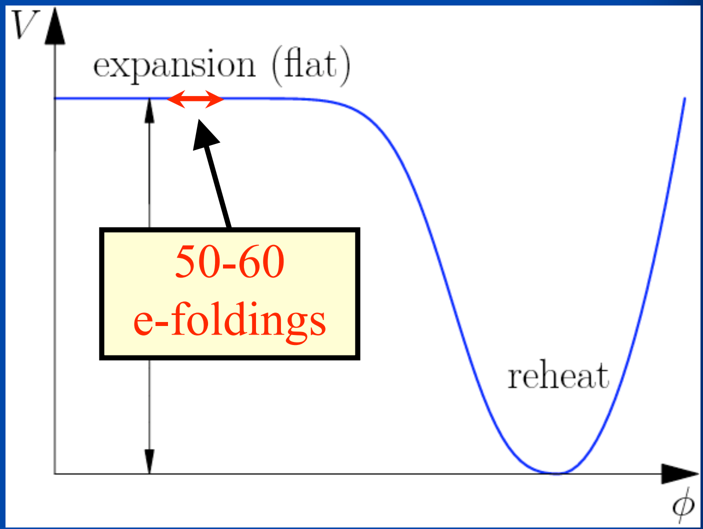
Spectrum of Perturbations

- Total number of inflation e-foldings $N_{tot} \geq 60$

- Spectrum of observable scales is produced
~ 50 – 60 e-foldings before the end of inflation

  - 50: later during inflation
    → smaller scales (~1 Mpc)

  - 60: earlier during inflation
    → larger scales (~3000 Mpc)



Prediction 2 of inflation is confirmed

- Multiple data sets (WMAP, large scale structure, etc) confirm n near 1.
- More detail shown in a minute to differentiate between models

Prediction 3 of Inflation:

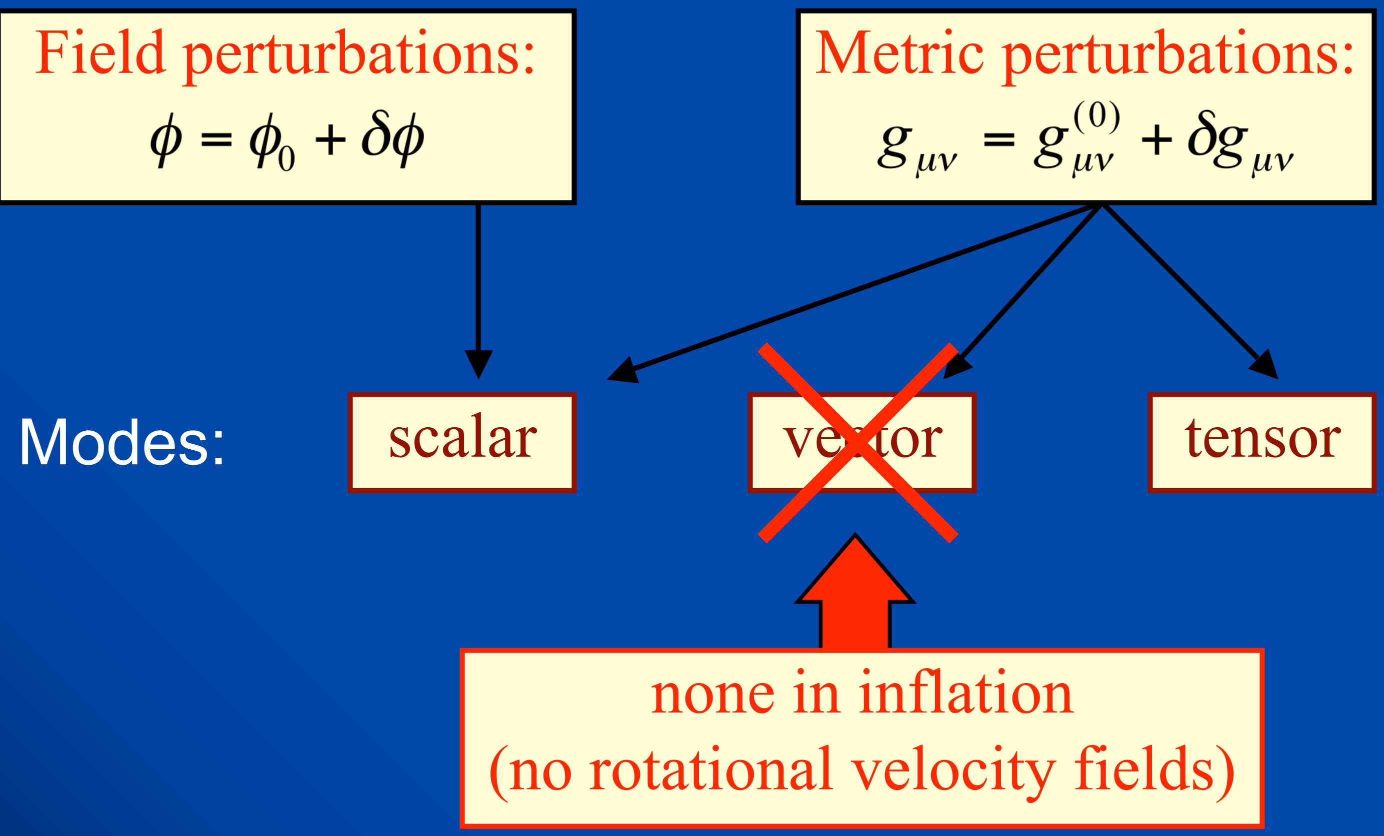
Existence of gravitational wave perturbations (tensor modes)

Prediction 3 of Inflation: Tensor (gravitational wave) modes

- In addition to density fluctuations, inflation also predicts the generation of tensor fluctuations with amplitude $P_T^{1/2} = \frac{H}{2\pi}$.
- For comparison with observation, the tensor amplitude is conventionally expressed as:
- $r = \frac{P_T^{1/2}}{P_\zeta^{1/2}}$ (denominator: scalar modes)

In principle there are four parameters describing the scalar and tensor fluctuations: the amplitudes and spectra of both components. The amplitude of the scalar perturbations is normalized by the height of the potential (the energy density Λ^4).The tensor spectral index n_T is not an independent parameter since it is related to the tensor/scalar ratio by the inflationary consistency condition $r = -8n_T$. The remaining free parameters are the spectral index n of the scalar density fluctuations, and the tensor amplitude (given by r).

Perturbations



Perturbations

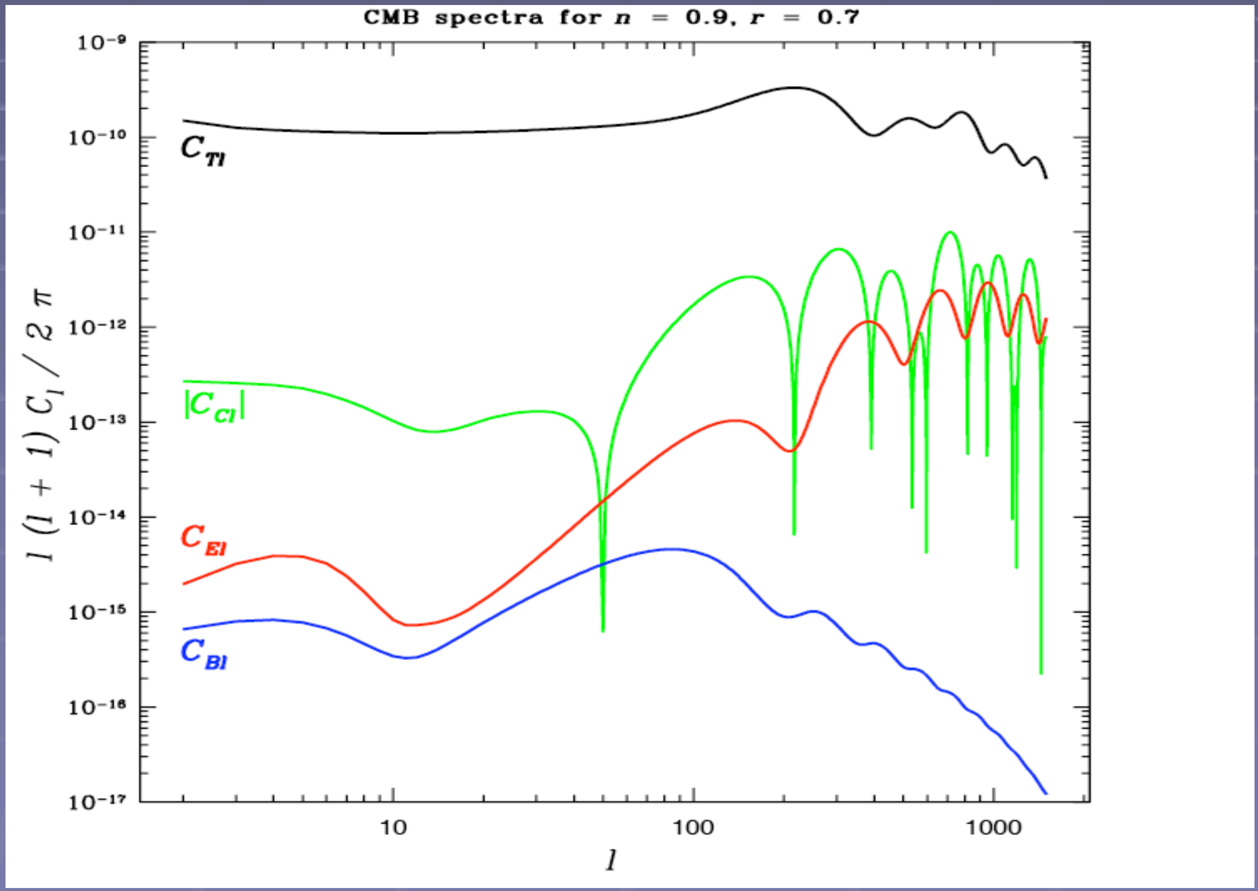
- Tensor modes $h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$
  - traceless transverse
  - 2 physical degrees of freedom (polarization)

- Gravitational wave modes $P_T^{1/2} = \frac{H}{2\pi}$

- B modes
  - Not yet detected
  - Can only arise from gravity waves

$\Rightarrow$ smoking gun

Gravity Modes are (at least) two orders of magnitude smaller than density fluctuations: hard to find!



Polarization

Temperature anisotropy, however, is not the whole story. The cosmic microwave background is also expected to be *polarized* due to the presence of fluctuations. Observation of polarization in the CMB will greatly increase the amount of information available for use in constraining cosmological models. Polarization is a *tensor* quantity, which can be decomposed on the celestial sphere into "electric-type", or scalar, and "magnetic-type", or pseudoscalar modes. The symmetric, trace-free polarization tensor $\mathcal{P}_{ab}$ can be expanded as [16]

$$\frac{\mathcal{P}_{ab}}{T_0} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[a_{lm}^{E} Y_{(lm)ab}^{E}(\theta, \phi) + a_{lm}^{B} Y_{(lm)ab}^{B}(\theta, \phi) \right], \tag{27}$$

where the $Y_{(lm)ab}^{E,B}$ are electric- and magnetic-type tensor spherical harmonics, with parity $(-1)^l$ and $(-1)^{l+1}$, respectively. Unlike a temperature-only map, which is described by the single multipole spectrum of C_l^T's, a temperature/polarization map is described by three spectra

$$\left\langle \left| a_{lm}^{T} \right|^2 \right\rangle \equiv C_{Tl}, \; \left\langle \left| a_{lm}^{E} \right|^2 \right\rangle \equiv C_{El}, \; \left\langle \left| a_{lm}^{B} \right|^2 \right\rangle \equiv C_{Bl}, \tag{28}$$

and three correlation functions,

$$\left\langle a_{lm}^{T*} a_{lm}^{E} \right\rangle \equiv C_{Cl}, \; \left\langle a_{lm}^{T*} a_{lm}^{B} \right\rangle \equiv C_{(TB)l}, \; \left\langle a_{lm}^{E*} a_{lm}^{B} \right\rangle \equiv C_{(EB)l}. \tag{29}$$

Parity requires that the last two correlation functions vanish, $C_{(TB)l} = C_{(EB)l} = 0$, leaving four spectra: temperature C_{Tl}, E-mode C_{El}, B-mode C_{Bl}, and the cross-correlation C_{Cl}. Figure 1 shows the four spectra for a typical case. Since scalar density perturbations have no "handedness," it is impossible for scalar modes to produce B-mode (pseudoscalar) polarization. Only tensor fluctuations (or foregrounds [54]) can produce a B-mode.

Four parameters from inflationary perturbations:

I. Scalar perturbations:

amplitude $(\delta\rho/\rho)|_s$ spectral index n_s

II. Tensor (gravitational wave) modes:

amplitude $(\delta\rho/\rho)|_T$ spectral index n_T

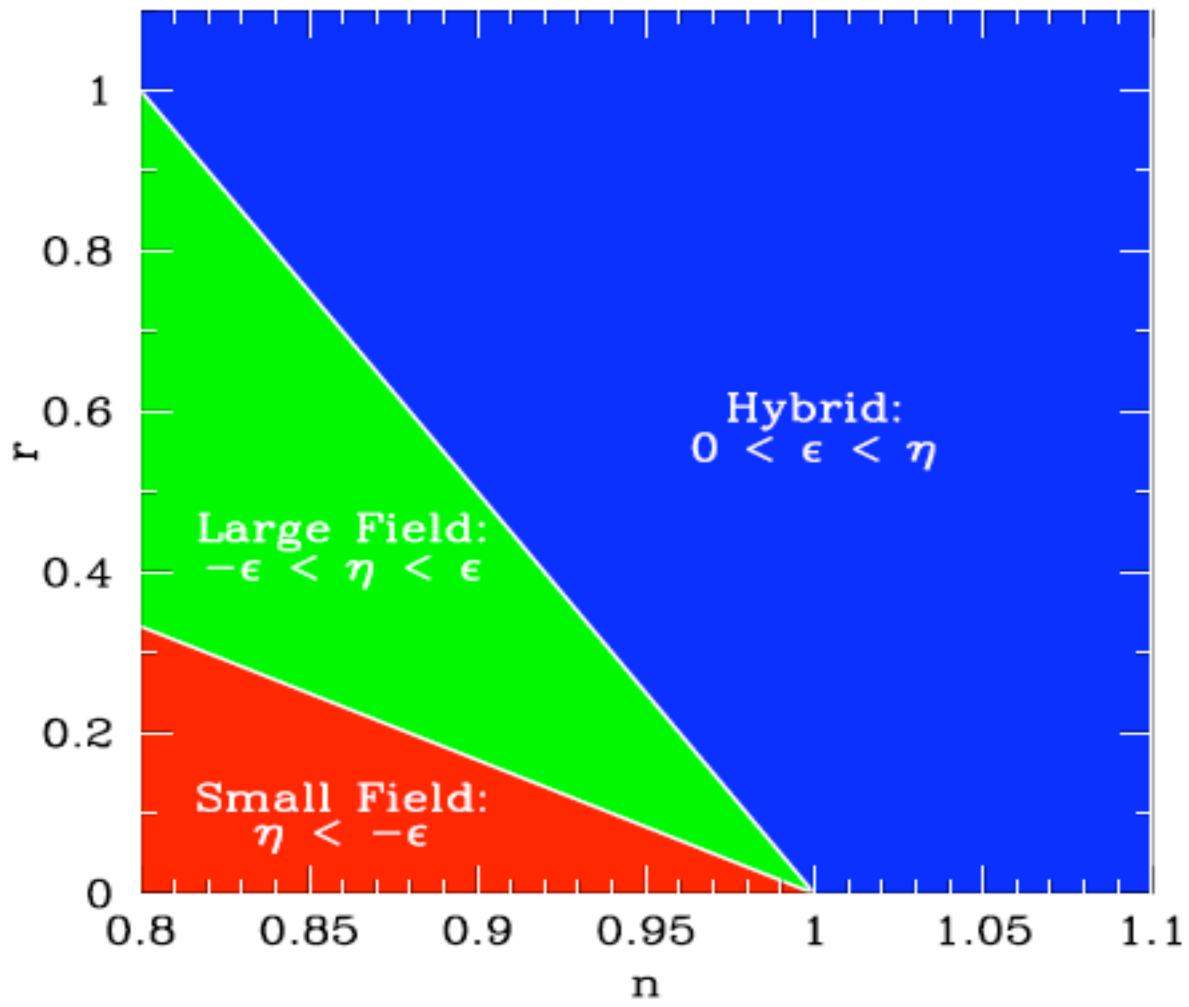
Expressed as

$$r \equiv \frac{P_T^{1/2}}{P_S^{1/2}}$$

Inflationary consistency condition: $r = -8n_T$

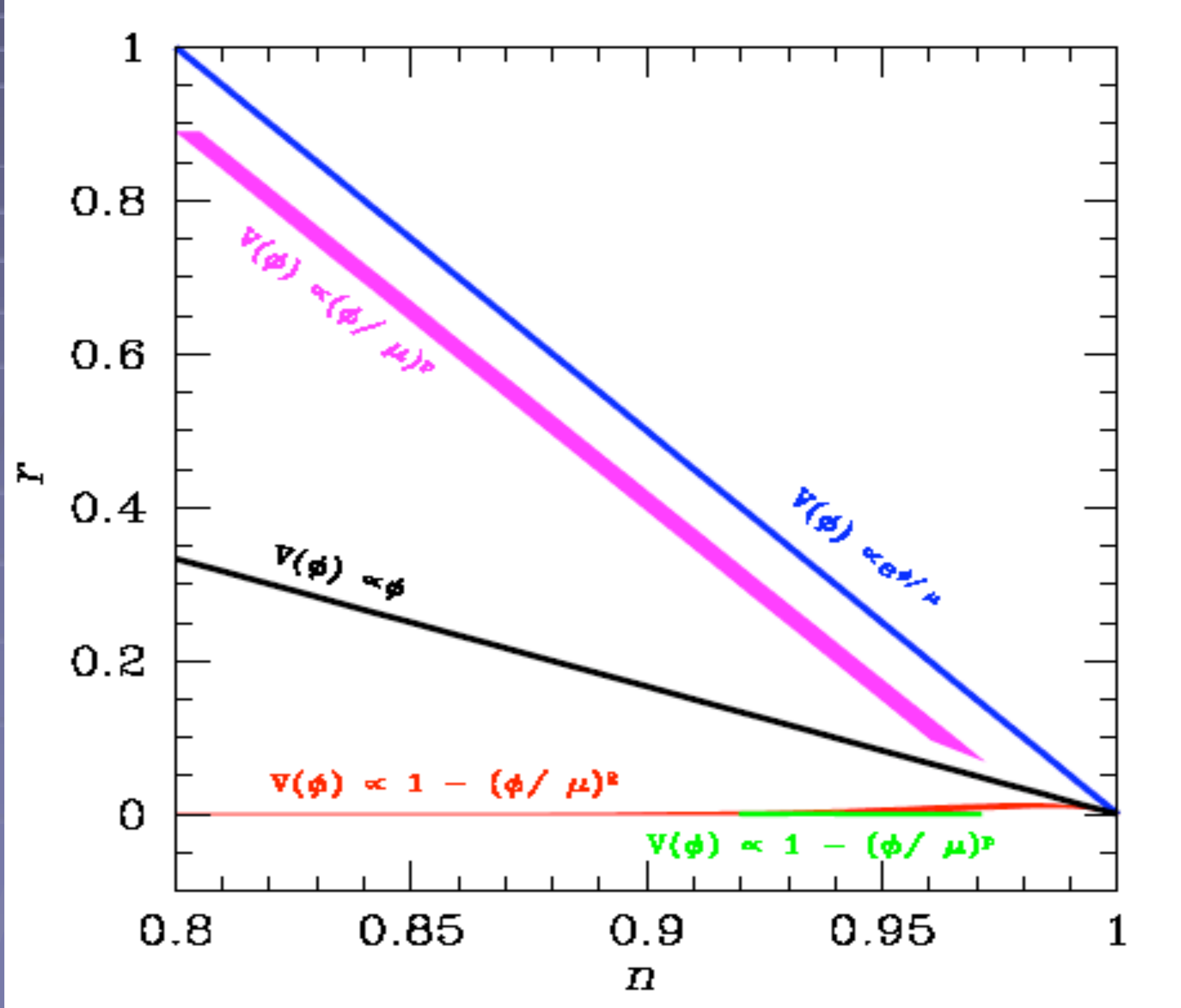
Plot in r-n plane

Different Types of Potentials in the r-n plane



(KINNEY 2002)

Examples of Models



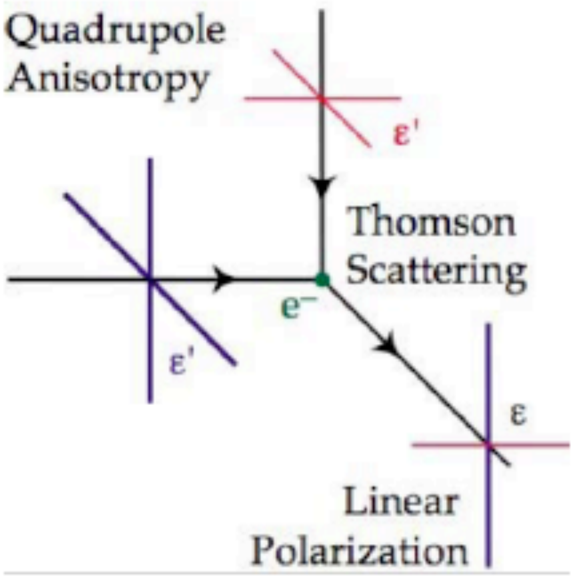
Sources of Polarization

Two ingredients

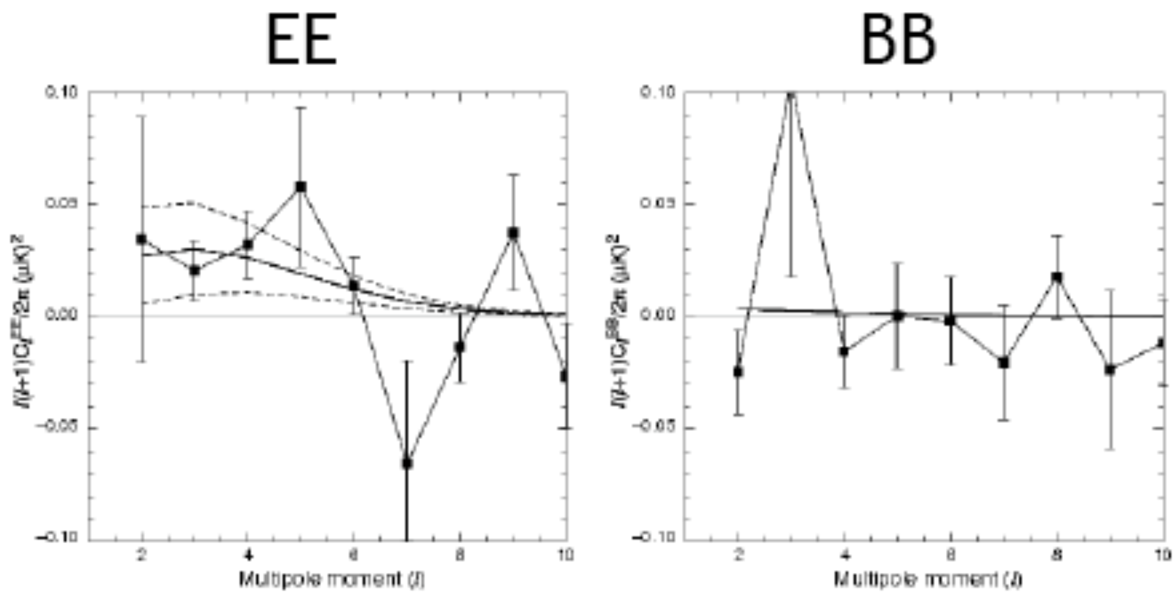
1. Free electrons
2. Incident quadrupole anisotropy

Scattering at z~1100 produces signal on degree scales

Scattering at z~10 produces signal on 10 degree scales - probes reionization from first stars.



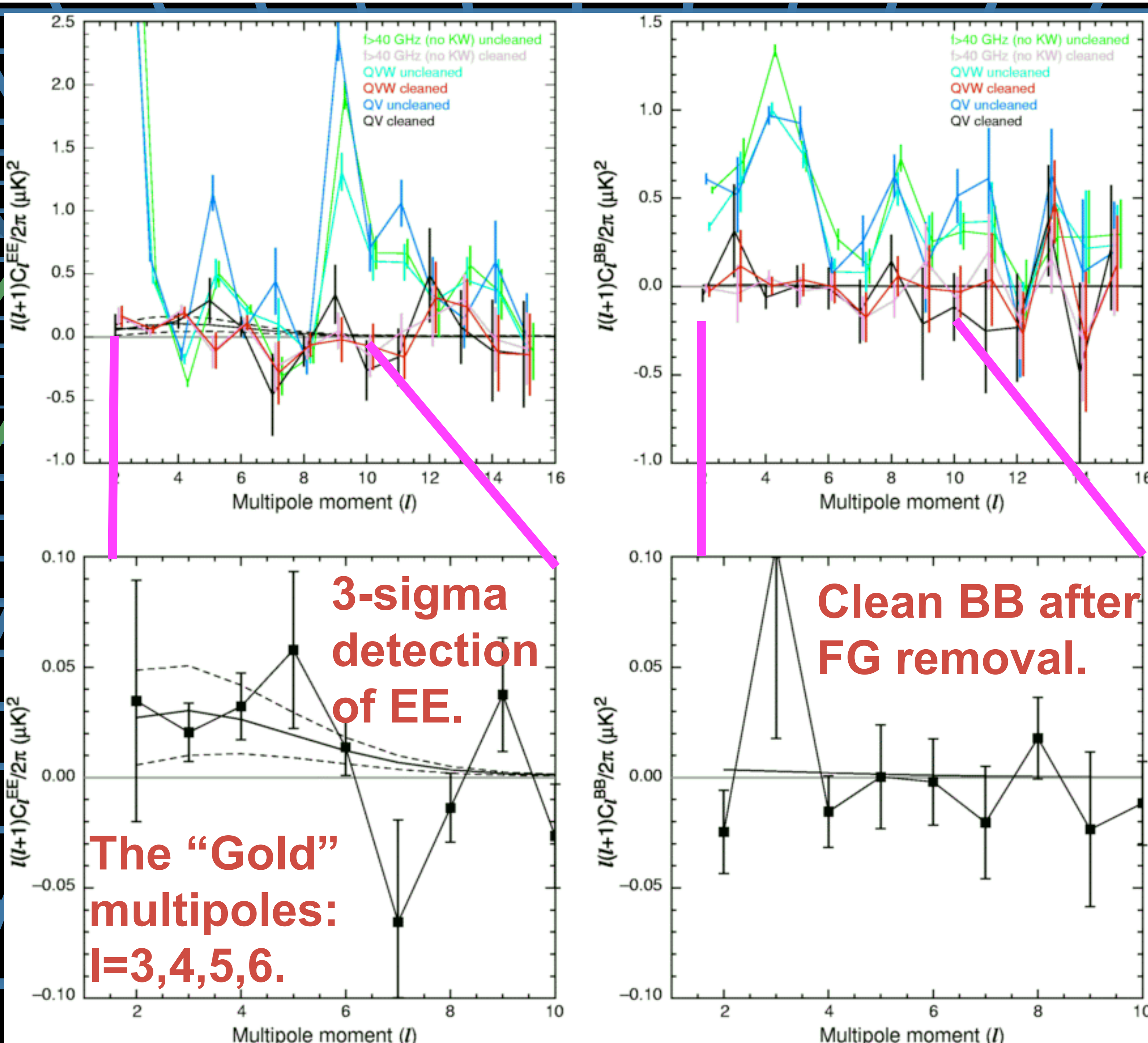
Best Estimate of Low-l Polarization Spectra



Data from 41 and 61 GHz only.

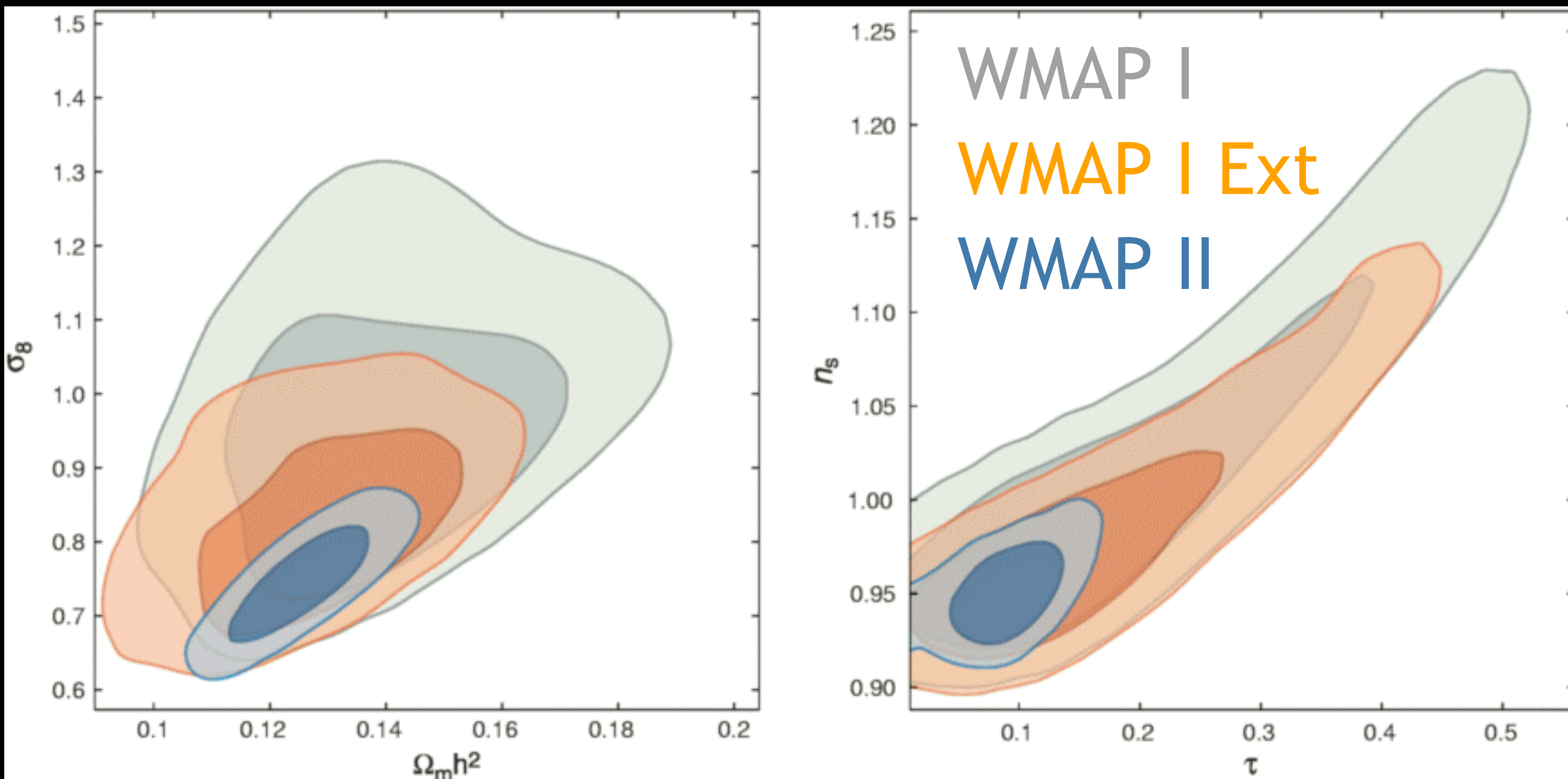
EE signal proportional to τ^2, provides new constraints on optical depth to last scattering surface (next slide).

BB consistent with zero.


f>40 GHz (no KW) uncleaned
f>40 GHz (no KW) cleaned
QVW uncleaned
QVW cleaned
QV uncleaned
QV cleaned
Multipole moment (l)
l(l+1)C_l^EE/2π (μK)^2
l(l+1)C_l^BB/2π (μK)^2
3-sigma detection of EE.
The “Gold” multipoles: l=3,4,5,6.
Clean BB after FG removal.

Effect of more data

LCDM model

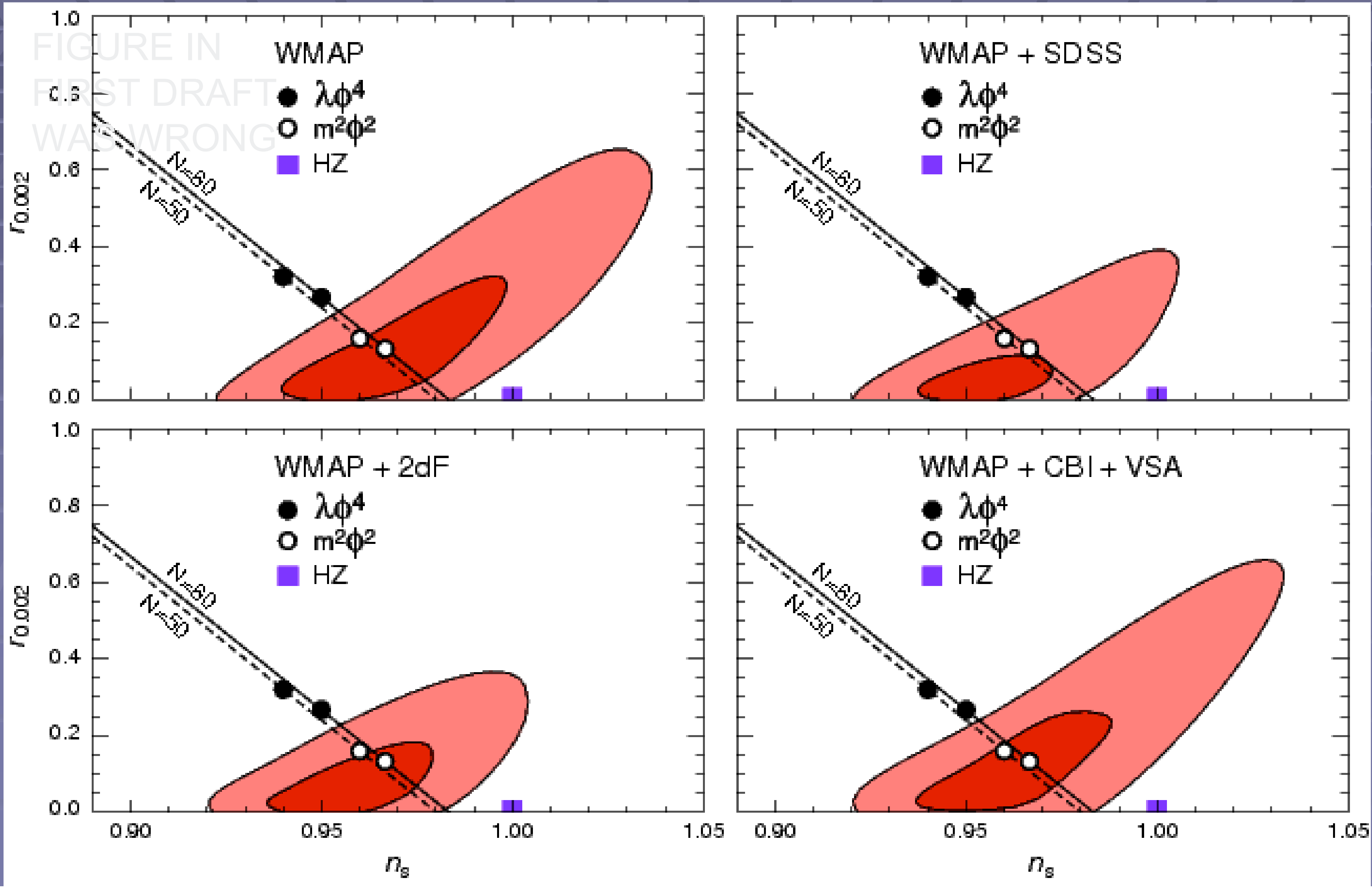


Reducing the noise by 3 → degeneracies broken

(taken from L. Verde)

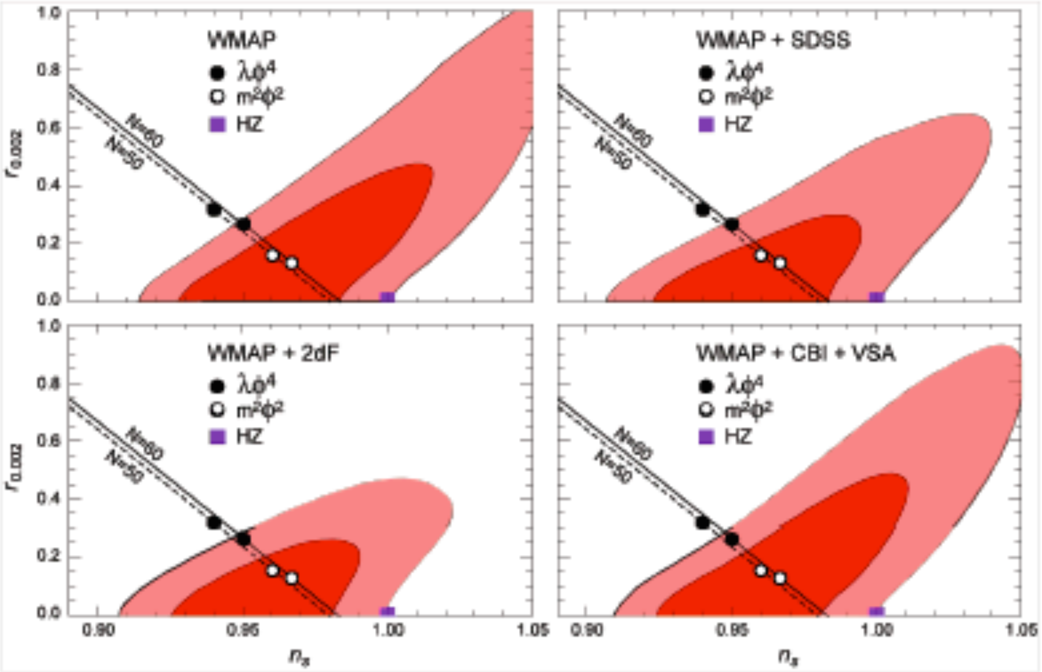
Tensor-to-scalar ratio r vs. scalar spectral index n

(FIGURE IN FIRST DRAFT WAS WRONG!)



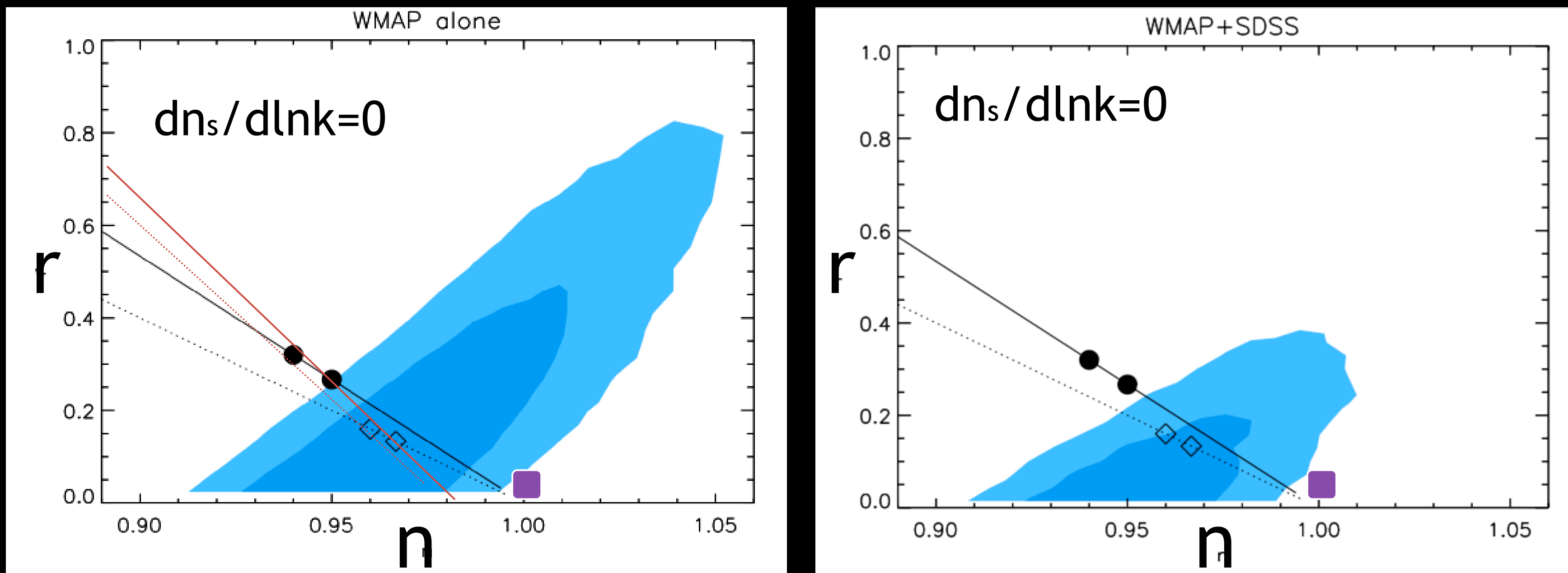
Tensor-to-scalar ratio r vs. scalar spectral index n

Testing Inflation with Tensors



Spectral index vs. tensors

Specific models critically tested



Models like $V(\phi) \sim \phi^p$

○ p=4 ◆ p=2 For 50 and 60 e-foldings

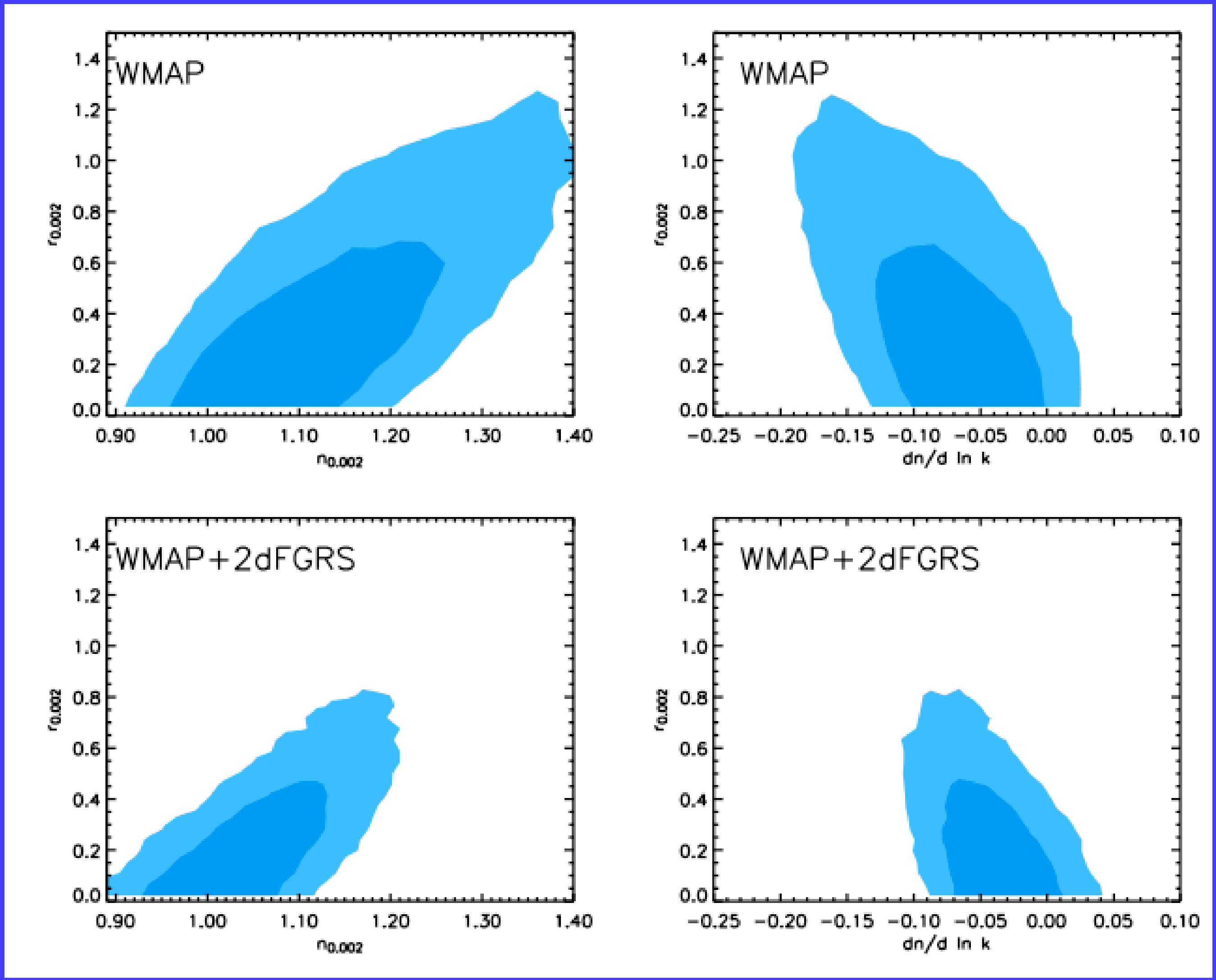
■ HZ

p fix, Ne varies

p varies, Ne fix

(taken from L. Verde)

The full treatment:



Where are we?

- General idea of inflation compares well to data: critical density, nearly scale invariant perturbations, superhorizon fluctuations.
- Now the data are becoming good enough to differentiate between models.
- Reconstructing the inflaton potential:

  Kolb, Lidsey, Abney, Copeland, Liddle 1995; Kinney, Kolb, Melchiorri, Riotto 2006; Alabidi and Lyth 2006

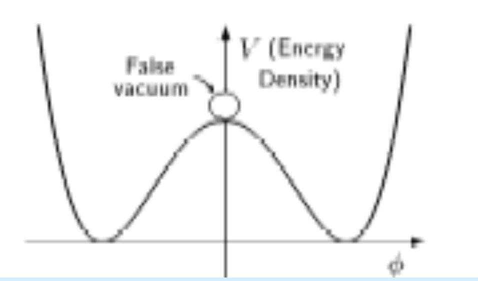
Natural Inflation after WMAP

- Theoretical motivation: no fine-tuning
- Recent interest in light of theoretical developments
- Unique predictions:
- Looks good compared to data

Katherine Freese
Christopher Savage

Fine Tuning in Rolling Models

- The potential must be very flat:



$$\frac{\Delta V}{(\Delta\Phi)^4} = \frac{\text{height}}{\text{width}^4} \leq 10^{-8},$$
$$e.g. V(\phi) = \lambda\Phi^4, \lambda \leq 10^{-12}$$

(Adams, Freese, and Guth 1990)

But particle physics typically gives this ratio = 1!

Inflationary Model Constraints

Success of inflationary models with rolling fields
$\Rightarrow$ constraints on $V(\phi)$

- Enough inflation

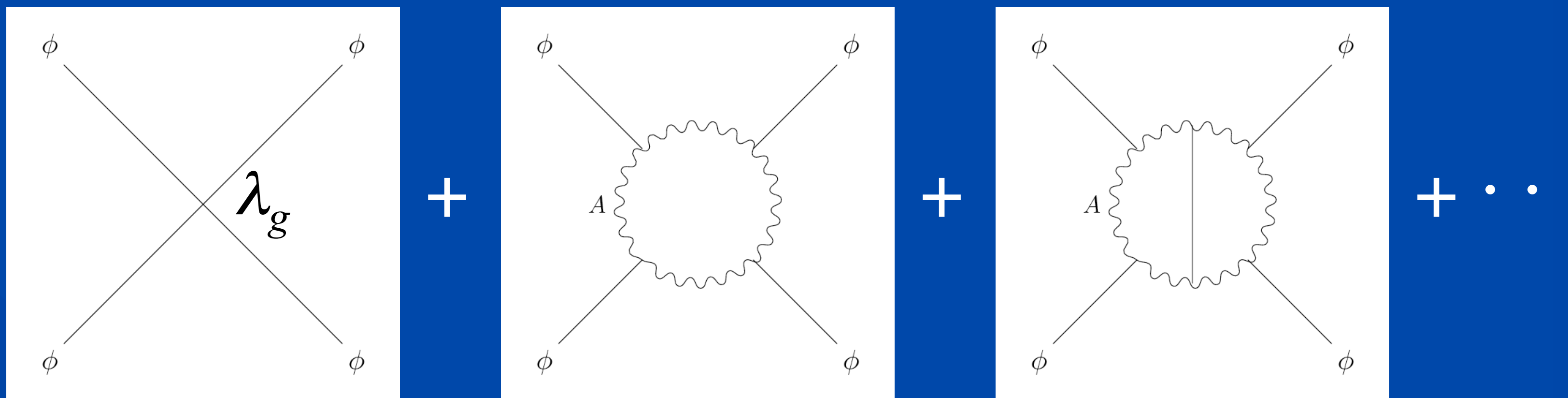
  Scale factor *a* must grow enough

$$\ln\left(\frac{a_{\text{end}}}{a_{\text{begin}}}\right) = \int_{t_{\text{begin}}}^{t_{\text{end}}} H\,dt = -8\pi G \int \frac{V(\phi)}{V'(\phi)}\,d\phi \geq 60$$

- Amplitude of density fluctuations not too large

$$\left.\frac{\delta\rho}{\rho}\right|_{\substack{\text{enter}\\ \text{horizon}}} \sim \left.\frac{H^2}{\dot{\phi}}\right|_{\substack{\text{exit}\\ \text{horizon}}} \leq \frac{\delta T}{T} \sim 10^{-5}$$

Fine Tuning due to Radiative Corrections



- Perturbation theory: 1-loop, 2-loop, 3-loop, etc.
- To keep $\lambda \sim 10^{-12}$ must balance tree level term against corrections to each order in perturbation theory. Ugly!

Inflation needs small ratio of mass scales

$$\frac{\Delta V}{(\Delta\Phi)^4} = \frac{\text{height}}{\text{width}^4} \leq 10^{-8},$$

- Two attitudes:

1) We know there is a heirarchy problem, wait until it’s explained

2) Two ways to get small masses in particles physics:

(i) supersymmetry

(ii) Goldstone bosons (shift symmetries)

Natural Inflation: Shift Symmetries

- Shift (axionic) symmetries protect flatness of inflaton potential

  $\Phi \longrightarrow \Phi + \text{constant}$ (inflaton is Goldstone boson)
- Additional explicit breaking allows field to roll.
- This mechanism, known as natural inflation, was first proposed in

Freese, Frieman, and Olinto 1990;
Adams, Bond, Freese, Frieman and Olinto 1993

Shift Symmetries

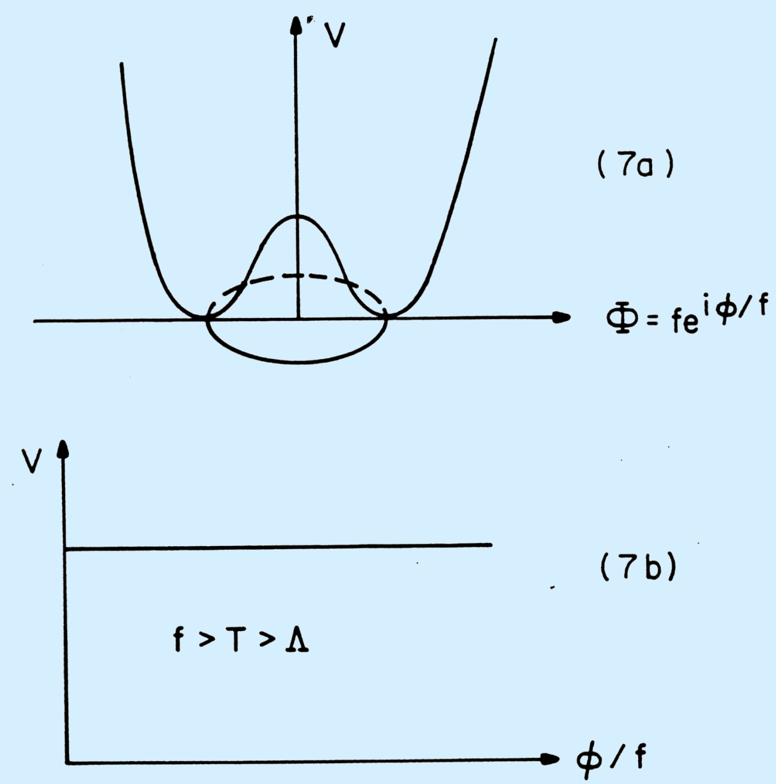
→ "Natural Inflation"
Freese, Frieman & Olinto (1990)

- We know of a particle with a small ratio of scales:
  the **axion**

$$\lambda_a \sim \left(\frac{\Lambda_{\text{QCD}}}{f_{\text{PQ}}}\right)^4 \sim 10^{-64}$$

- IDEA: use a potential similar to that for axions in inflation
  ⇒ natural inflation (no fine-tuning)
  - Here, we do *not* use the QCD axion.
    We use a heavier particle with similar behavior.

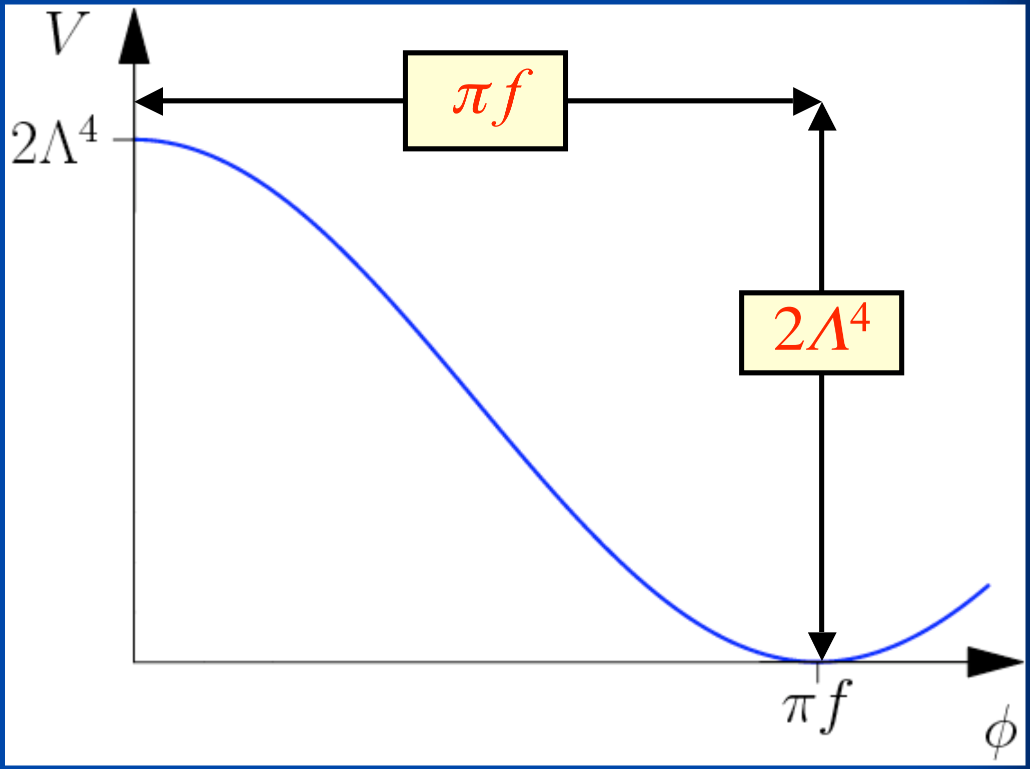
e.g., mimic the physics of the axion (Weinberg; Wilczek)



Natural Inflation

For QCD axion:
$f \sim 10^{12}$ GeV
$\Lambda \sim 100$ MeV

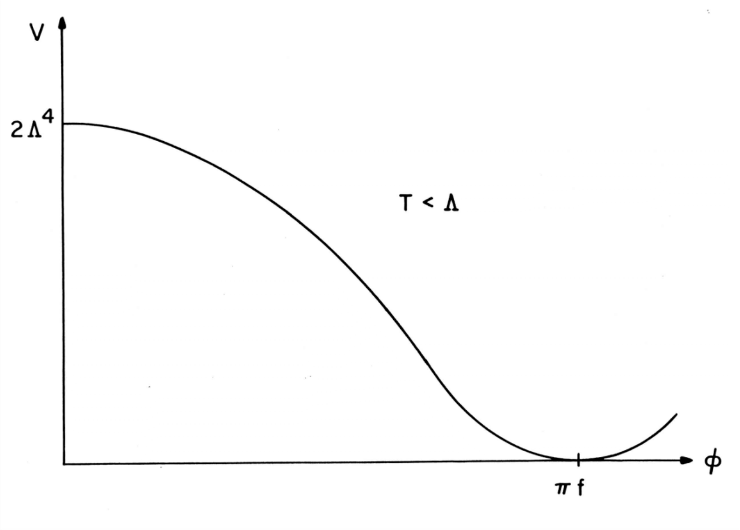
For natural inflation:
$f \sim M_{\rm Pl}$
$\Lambda \sim M_{\rm GUT}$



- Width f :
  Scale of spontaneous symmetry breaking of some global symmetry
- Height Λ:
  Scale at which gauge group becomes strong

Natural Inflation

(Freese, Frieman, and Olinto 1990;
Adams, Bond, Freese, Frieman and Olinto 1993)



$$V(\Phi) = \Lambda^4[1 + \cos(\Phi/f)]$$

- Two different mass scales:
- Width f is the scale of SSB of some global symmetry
- Height Λ is the scale at which some gauge group becomes strong

Two Mass Scales Provide required heirarchy

- For QCD axion,

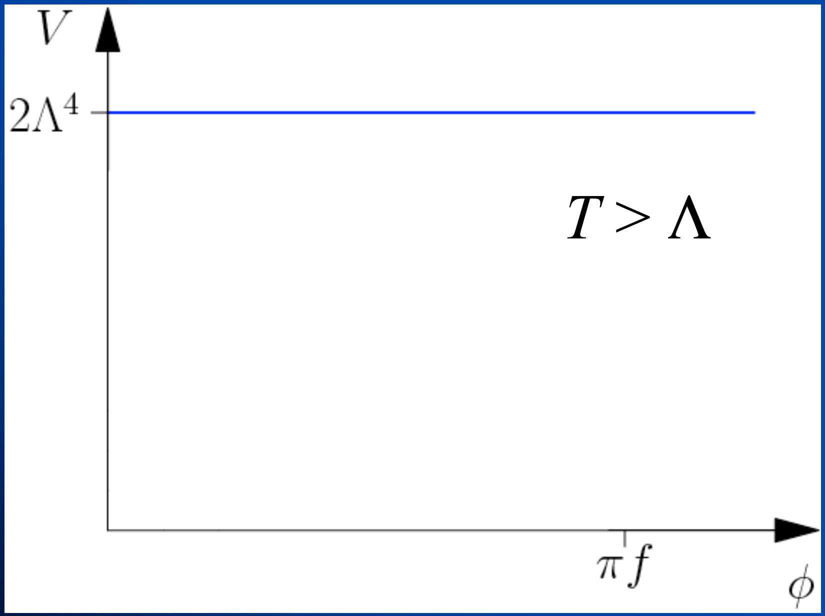
$$\Lambda_{\mathrm{QCD}} \sim 100\mathrm{MeV}, f_{PQ} \sim 10^{12}\mathrm{GeV}, \frac{\mathrm{height}}{\mathrm{width}} \sim 10^{-64}!!$$

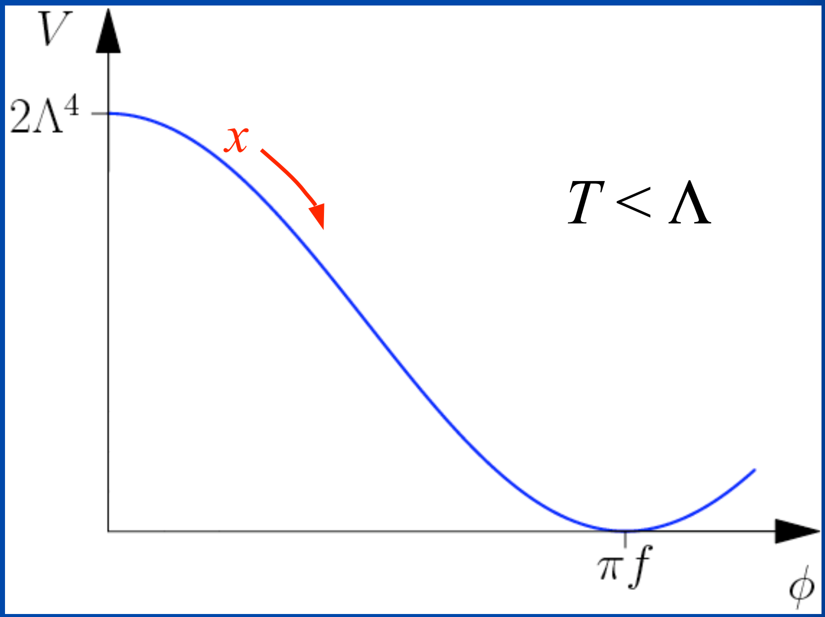
- For inflation, need $\Lambda \sim m_{GUT}, f \sim m_{pl}$

Enough inflation requires width = f ≈ mpl,
Amplitude of density fluctuations requires
height = $\Lambda \sim m_{GUT}$

Sufficient Inflation

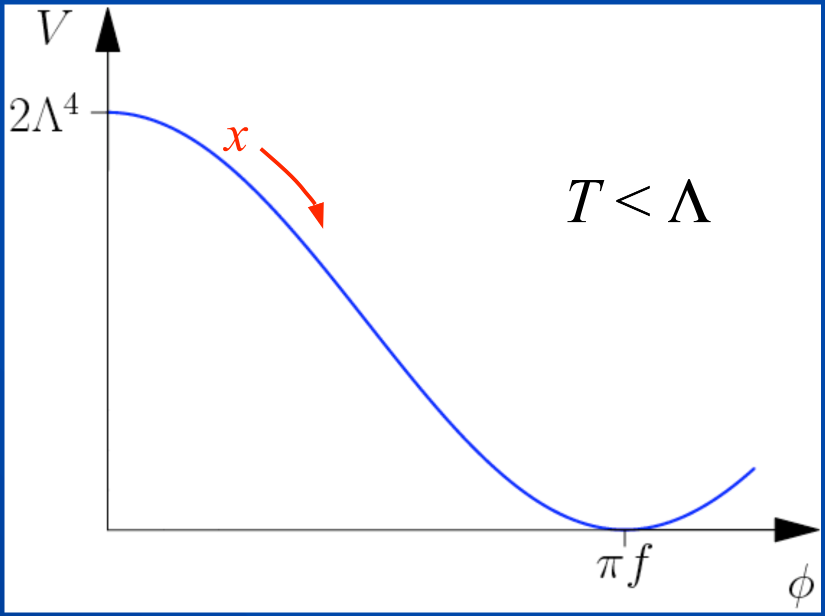
- ϕ initially randomly distributed between 0 and πf at different places in the universe.
- $T < \Lambda$: ϕ rolls down the hill. The pieces of the universe with ϕ far enough uphill will inflate enough.





Sufficient Inflation

- ϕ rolls down the hill.
  The pieces of the universe with ϕ far enough uphill will inflate enough.



Sufficient Inflation

- *A posteriori* probability:
  Those pieces of the universe that do inflate end up very large. Slice the universe after inflation and see what was probability of sufficient inflation.
  - Numerically evolved scalar field

$$N \equiv \ln\left(\frac{a_2}{a_1}\right) = \int H\,dt = \frac{-8\pi}{M_{\rm Pl}^2}\int \frac{V}{V'}\,d\phi = \frac{16\pi f^2}{M_{\rm Pl}^2}\ln\left[\frac{\sin(\phi_2/2f)}{\sin(\phi_1/2f)}\right] \geq 60$$

$$P = 1 - \frac{\int_{\phi_1^{\max}}^{\pi f} d\phi_1 \exp[3N(\phi_1)]}{\int_{H/2\pi}^{\pi f} d\phi_1 \exp[3N(\phi_1)]}$$

For $f \geq 0.06\, M_{\rm Pl}$,
P = $O(1)$

Density Fluctuations

Largest at 60 efolds before end of inflation

$$\frac{\delta\rho}{\rho} \approx \frac{H^2}{\dot{\phi}} \approx \frac{3\Lambda^2 f}{M_{\rm Pl}^3} \frac{\left[1+\cos(\phi_1^{\max}/f)\right]^{3/2}}{\sin(\phi_1^{\max}/f)} \sim 10^{-5}$$

$\Rightarrow \Lambda \sim 10^{15}$ GeV – 10^{16} GeV (height of potential)

$\Rightarrow m_\phi = \Lambda^2/\phi \sim 10^{11}$ GeV – 10^{13} GeV

- Density fluctuation spectrum is non-scale invariant with extra power on large length scales

$$P_k = |\delta_k|^2 \sim k^{n_s}$$

$$\text{with} \quad n_s \approx 1 - \frac{M_{\rm Pl}^2}{8\pi f^2} \quad (\text{for } f < M_{\rm Pl})$$

WMAP $\Rightarrow$ f > 0.7 $M_{\rm PL}$

Implementations of natural inflation's shift symmetry

- Natural chaotic inflation in SUGRA using shift symmetry in Kahler potential (Gaillard, Murayama, Olive 1995; Kawasaki, Yamaguchi, Yanagida 2000)
- In context of extra dimensions: Wilson line with (Arkani-Hamed et al 2003) but Banks et al (2003) showed it fails in string theory. $f \gg m_{pl}$
- "Little" field models (Kaplan and Weiner 2004)
- In brane Inflation ideas (Firouzjahi and Tye 2004)
- Gaugino condensation in SU(N) $\times$ SU(M):

Adams, Bond, Freese, Frieman, Olinto 1993;

Blanco-Pillado et al 2004 (Racetrack inflation)

Legitimacy of large axion scale?

Natural Inflation needs $f > 0.6 m_{pl}$

Is such a high value compatible with an effective field theory description? Do quantum gravity effects break the global axion symmetry?

Kinney and Mahantappa 1995: symmetries suppress the mass term and $f \ll m_{pl}$ is OK.

Arkani-Hamed et al (2003):axion direction from Wilson line of U(1) field along compactified extra dimension provides $f \gg m_{pl}$

However, Banks et al (2003) showed it does not work in string theory.

A large effective axion scale

(Kim, Nilles, Peloso 2004)

- Two or more axions with low PQ scale can provide large $f_{eff} \sim m_{pl}$
- Two axions $\theta \text{ and } \rho$

$$V = \Lambda_1^4\left[1-\cos\left(\frac{\theta}{f}+\frac{\epsilon_1\rho}{g}\right)\right]+\Lambda_2^4\left[1-\cos\left(\frac{\theta}{f}+\frac{\epsilon_2\rho}{g}\right)\right]$$

- Mass eigenstates are linear combinations of $\theta \text{ and } \rho$
- Effective axion scale can be large,

$$f_\xi = \frac{\sqrt{\epsilon_1^2 f^2 + g^2}}{\epsilon_1 - \epsilon_2} >> f \;\; \text{if} \;\; |\epsilon_1 - \epsilon_2| << 1$$

Also, N-flation has a large number of axions (Dimopolous et al 2005))

Density Fluctuations and Tensor Modes can determine which model is right

- Density Fluctuations:

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}}, \quad |\delta_k|^2 \sim k^{n_s}$$

WMAP data:

$$|n_s - 1| < 0.1$$

Slight indication of running of spectral index

- Tensor Modes $P_T^{1/2} = \frac{H}{2\pi}$ gravitational wave modes, detectable in upcoming experiments

Density Fluctuations in Natural Inflation

- Power Spectrum:

$$|\delta_k|^2 \sim k^{n_s}, n_s = 1 - \frac{m_{pl}^2}{8\pi f^2}$$

- WMAP data:

implies $|n_s - 1| < 0.1$

$$f \geq 0.6 m_{pl}$$

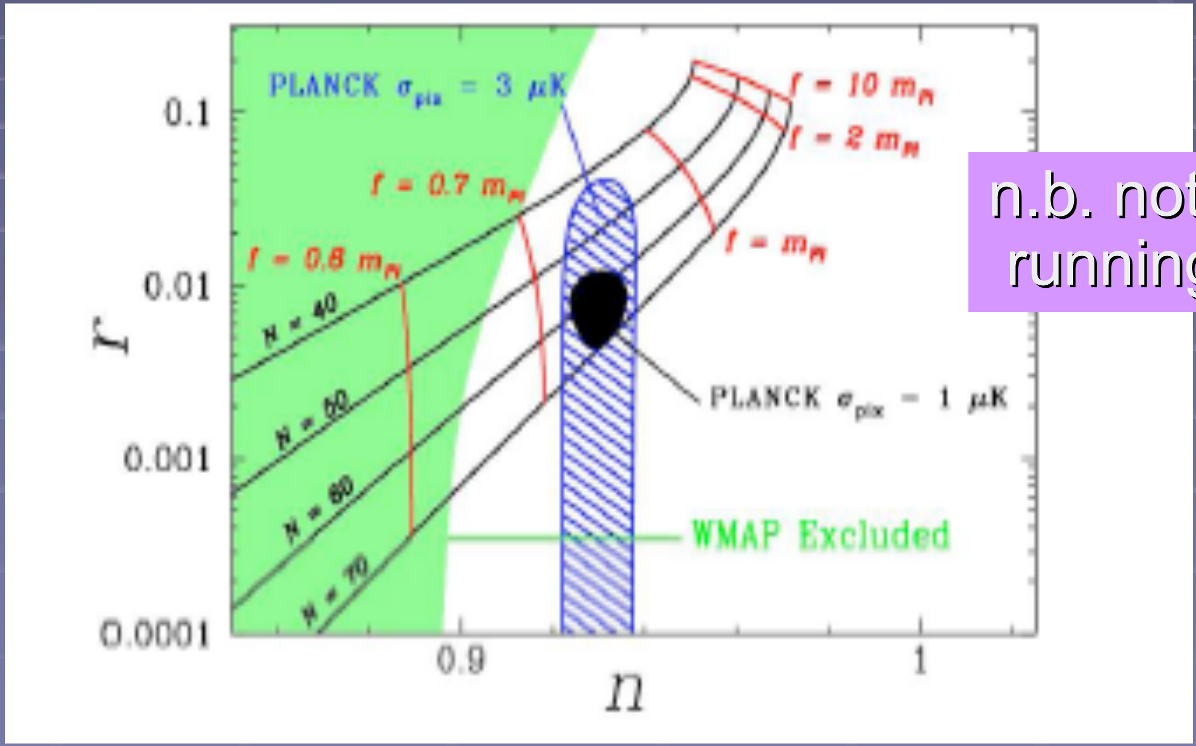
(Freese and Kinney 2004)

Tensor Modes in Natural Inflation

(original model) (Freese and Kinney 2004)

Two predictions, testable in next decade: 1) Tensor modes, while smaller than in other models, must be found. 2) There is very little running of n in natural inflation.

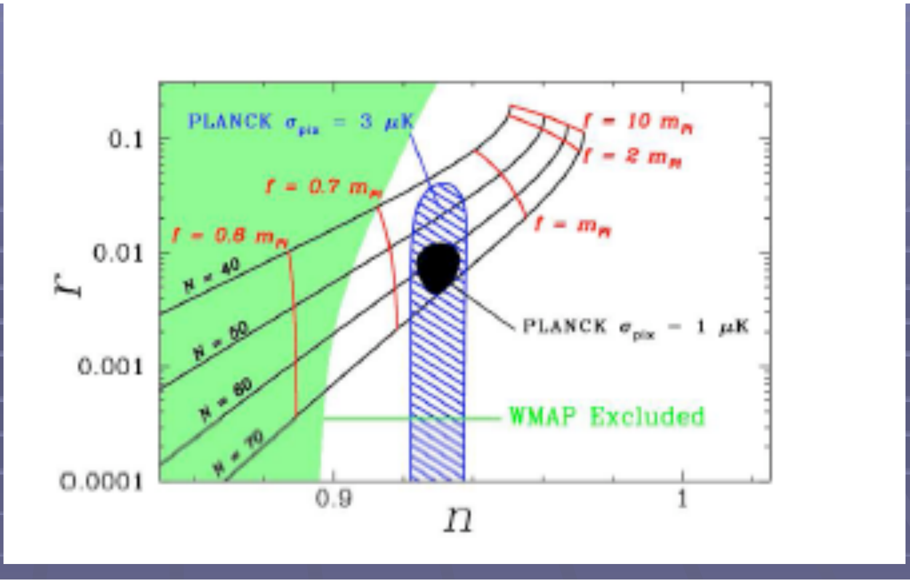
$$r = \frac{P_T^{1/2}}{P_s^{1/2}} = 16\epsilon$$

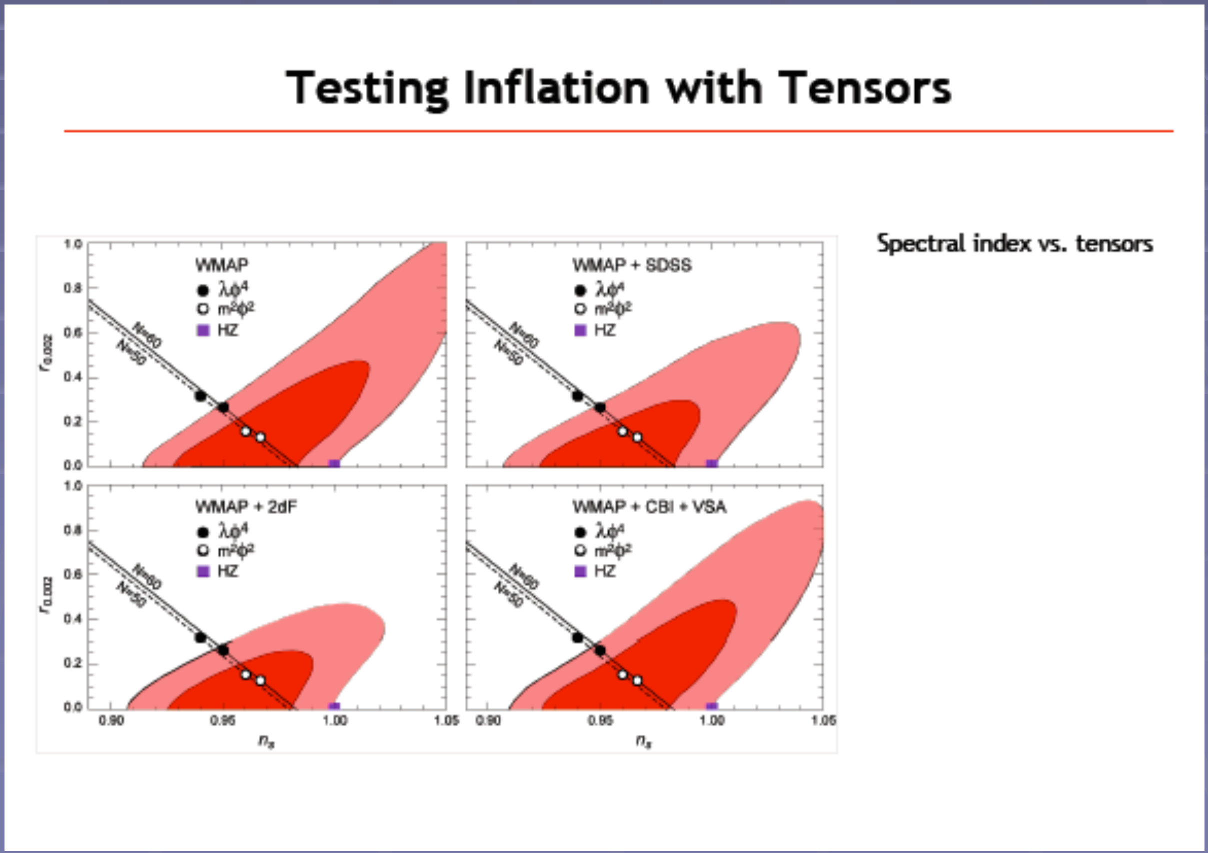


n.b. not much running of n

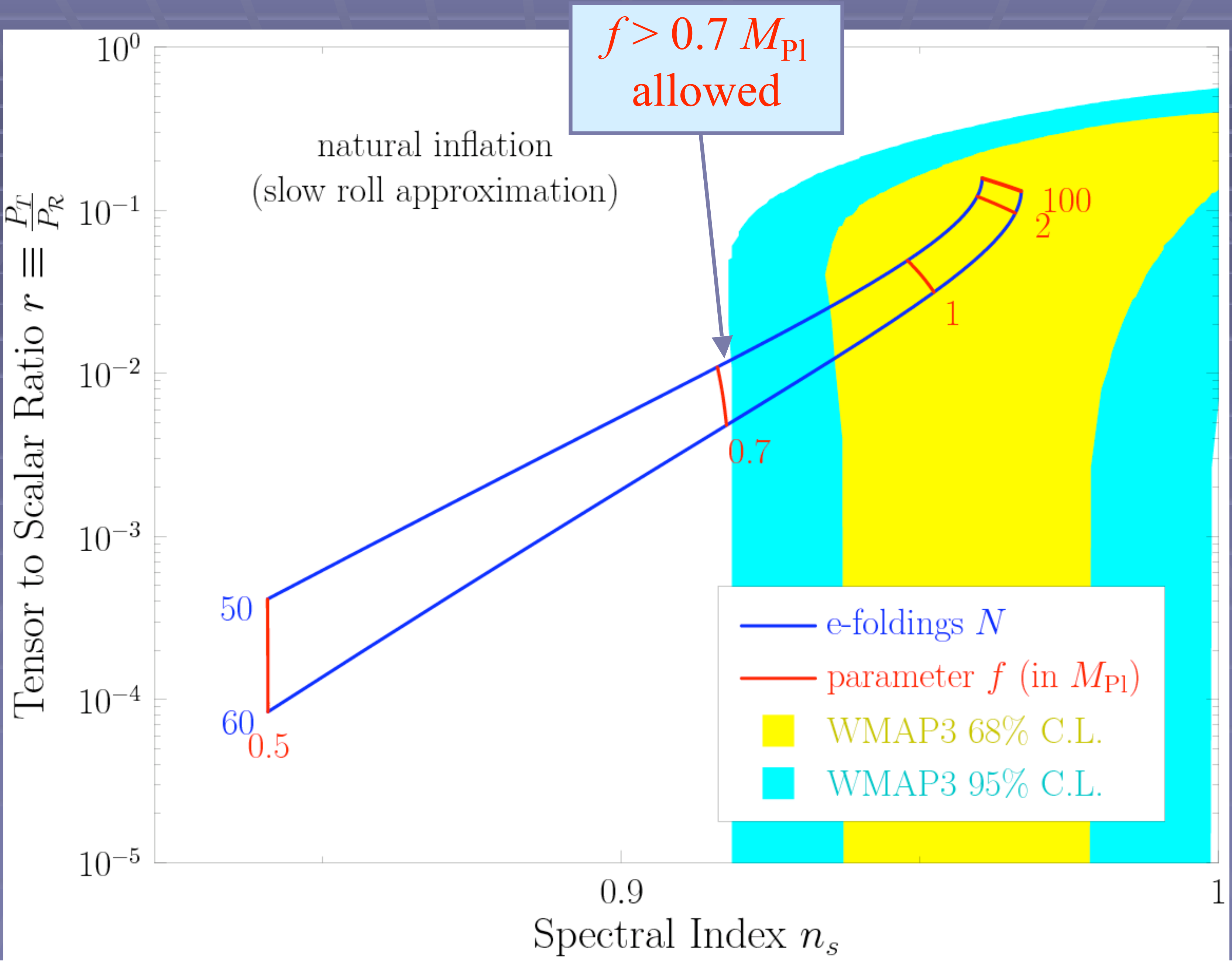
Sensitivity of PLANCK: error bars +/- 0.05 on r and 0.01 on n. Next generation expts (3 times more sensitive) must see it.

Natural Inflation agrees well with WMAP!

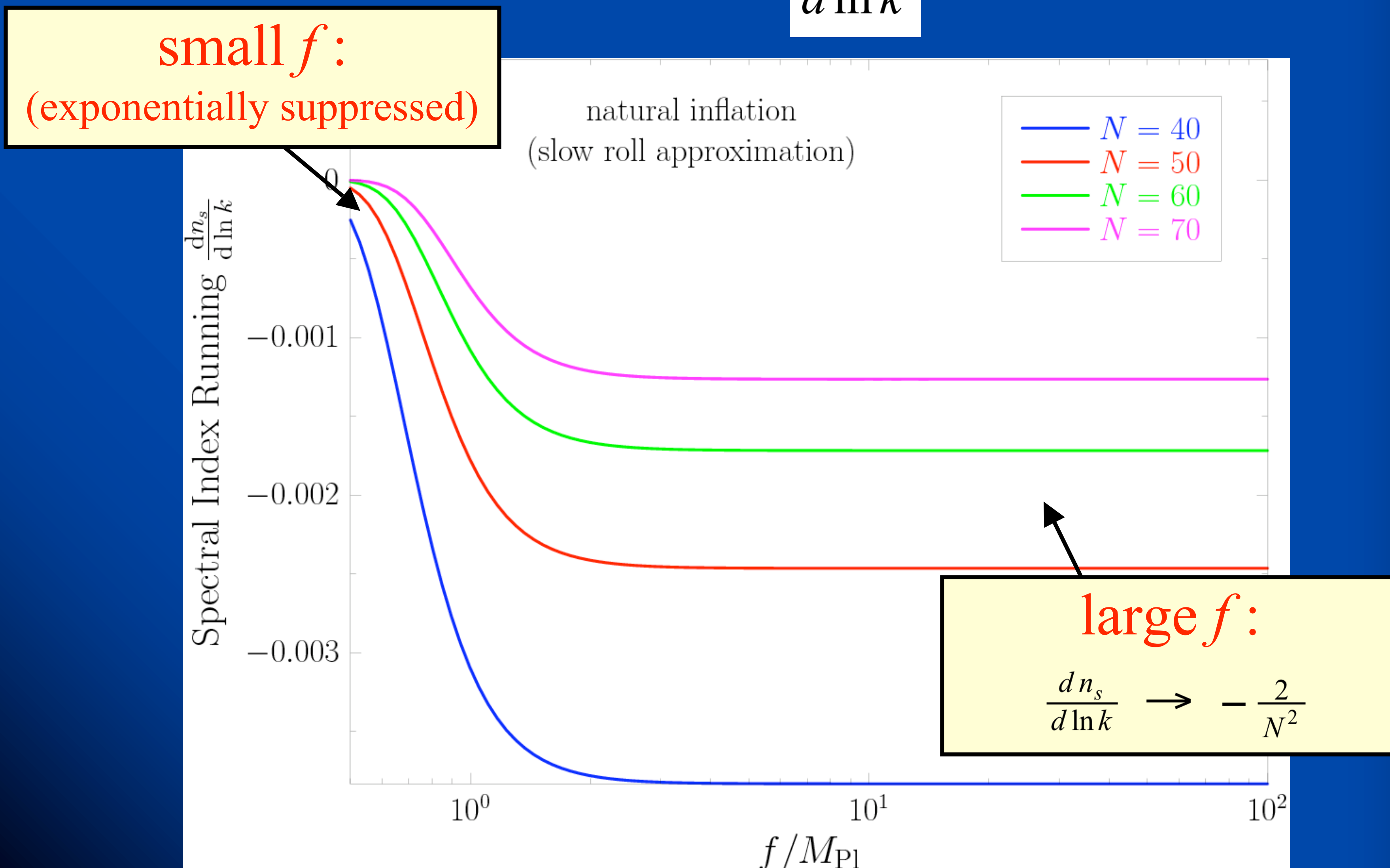




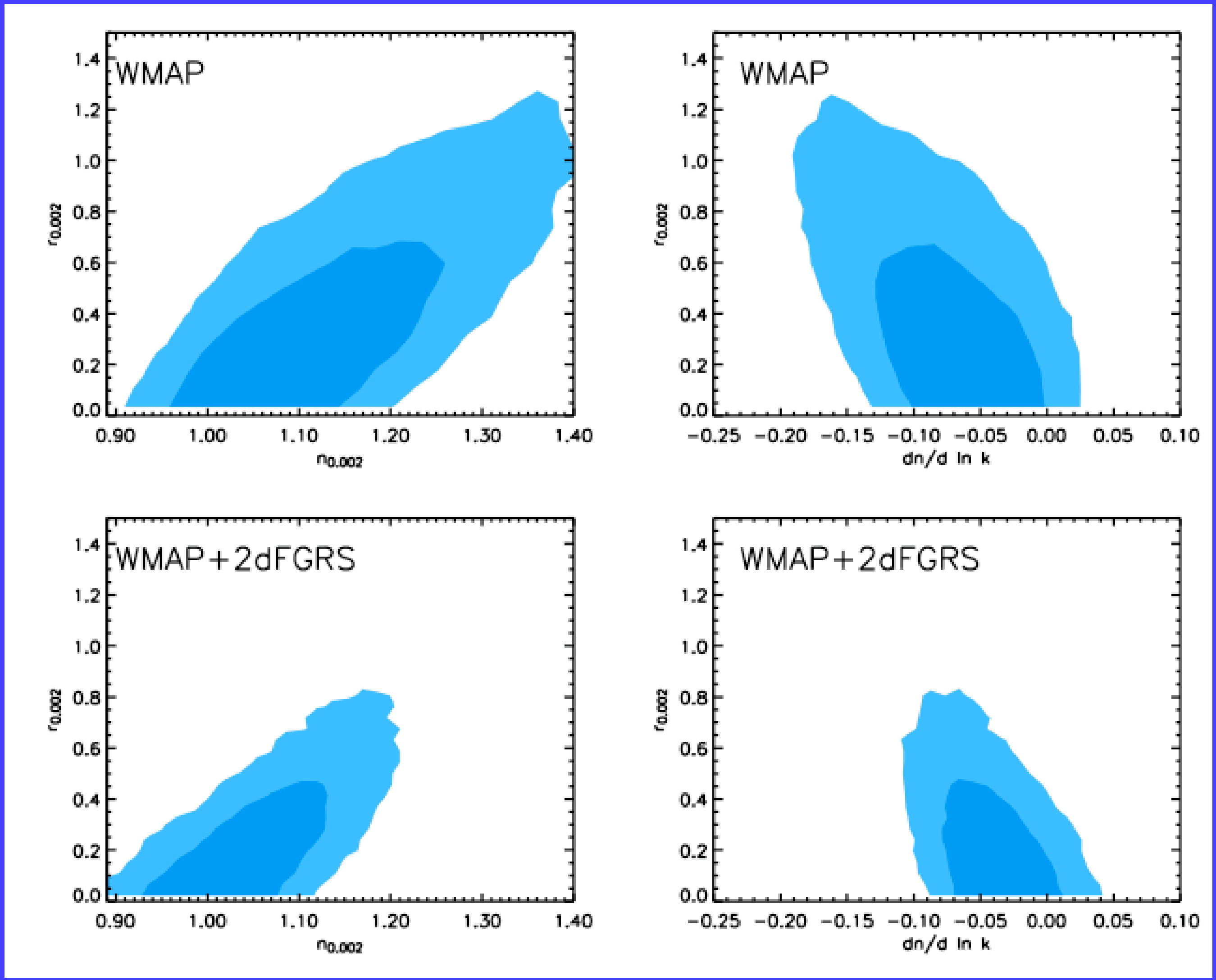
r-n plane: Natural inflation after WMAP 3



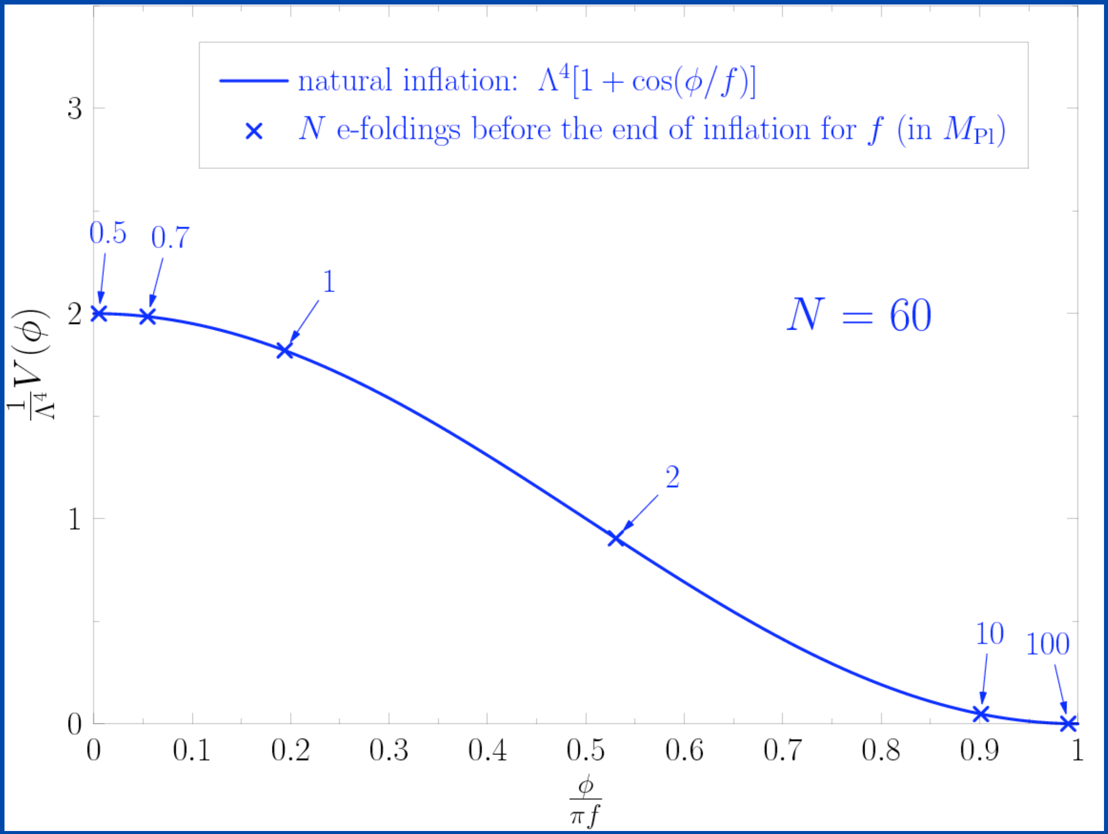
Spectral Index Running $\frac{d n_s}{d \ln k}$



The full treatment:

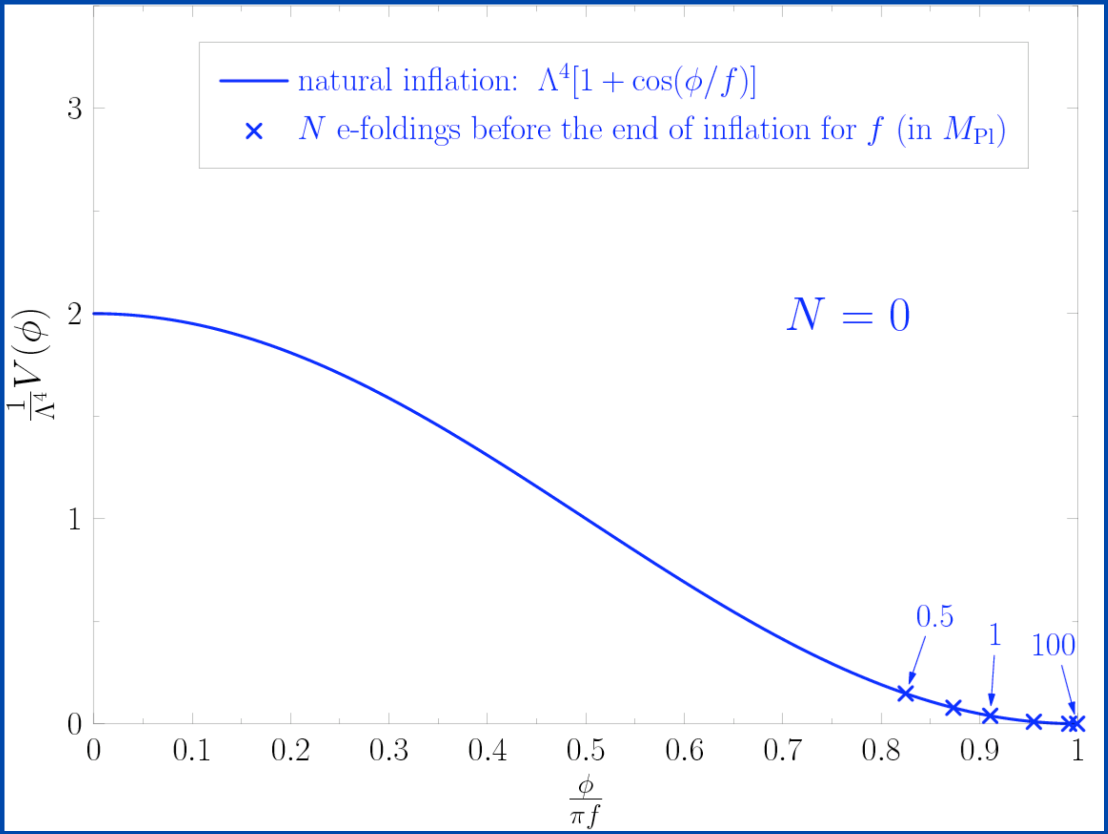


Potential



- 60 e-foldings before the end of inflation
  ~ present day horizon

Potential

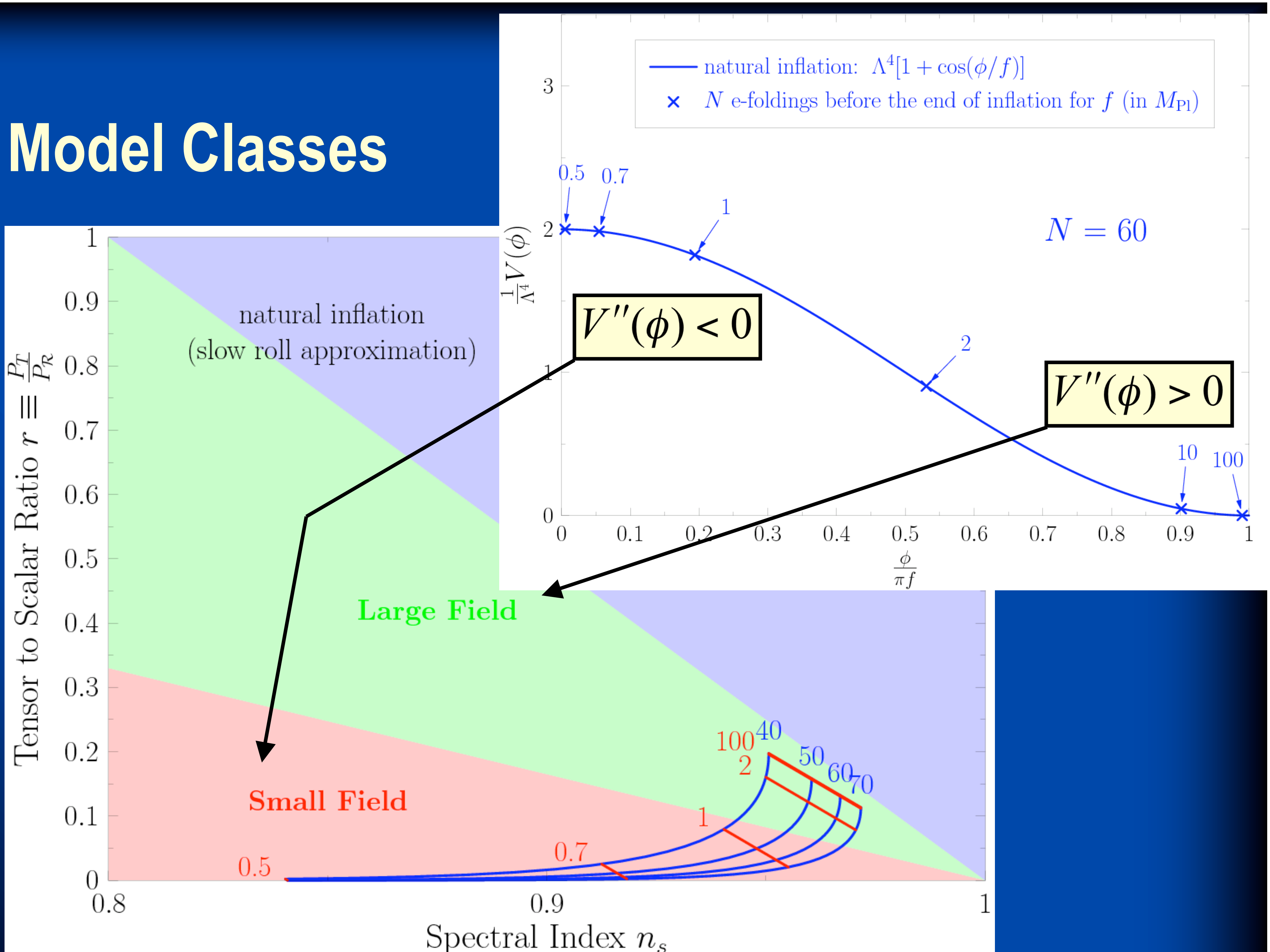


- At the end of inflation

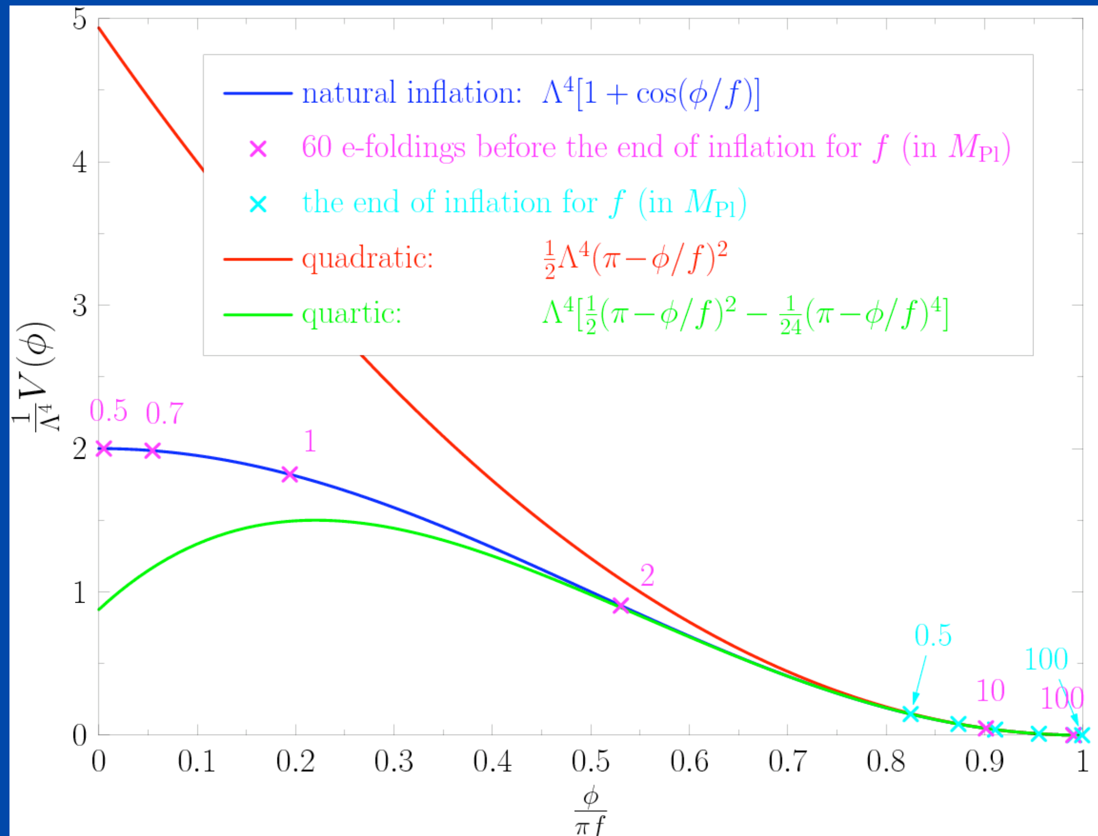
Model Classes

- Kinney & collaborators
  - Large-field $-\varepsilon < \eta \leq \varepsilon$ $V''(\phi) > 0$
  - Small-field $\eta < -\varepsilon$ $V''(\phi) < 0$
  - Hybrid $0 < \varepsilon < \eta$

Model Classes



Potential



- f > few M_{pl}: V(ϕ) ~ quadratic

Natural Inflation Summary

- No fine tuning,
  naturally flat potential

- WMAP 3-year data:

  $f < 0.7\ M_{\mathrm{Pl}}$ excluded

  $f > 0.7\ M_{\mathrm{Pl}}$ consistent
  - Tensor/scalar ratio r
  - Spectral index n_s
  - Spectral index running $\mathrm{d}n_s/\mathrm{d}\ln k$

To really test inflation need B modes, which can only be produced by gravity waves.

- Will confirm key prediction of inflation.
- Will differentiate between models.
- Need next generation experiments.

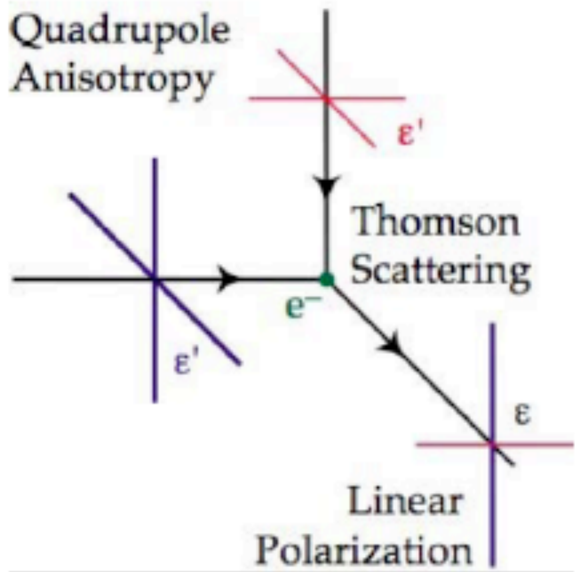
Sources of Polarization

Two ingredients

1. Free electrons
2. Incident quadrupole anisotropy

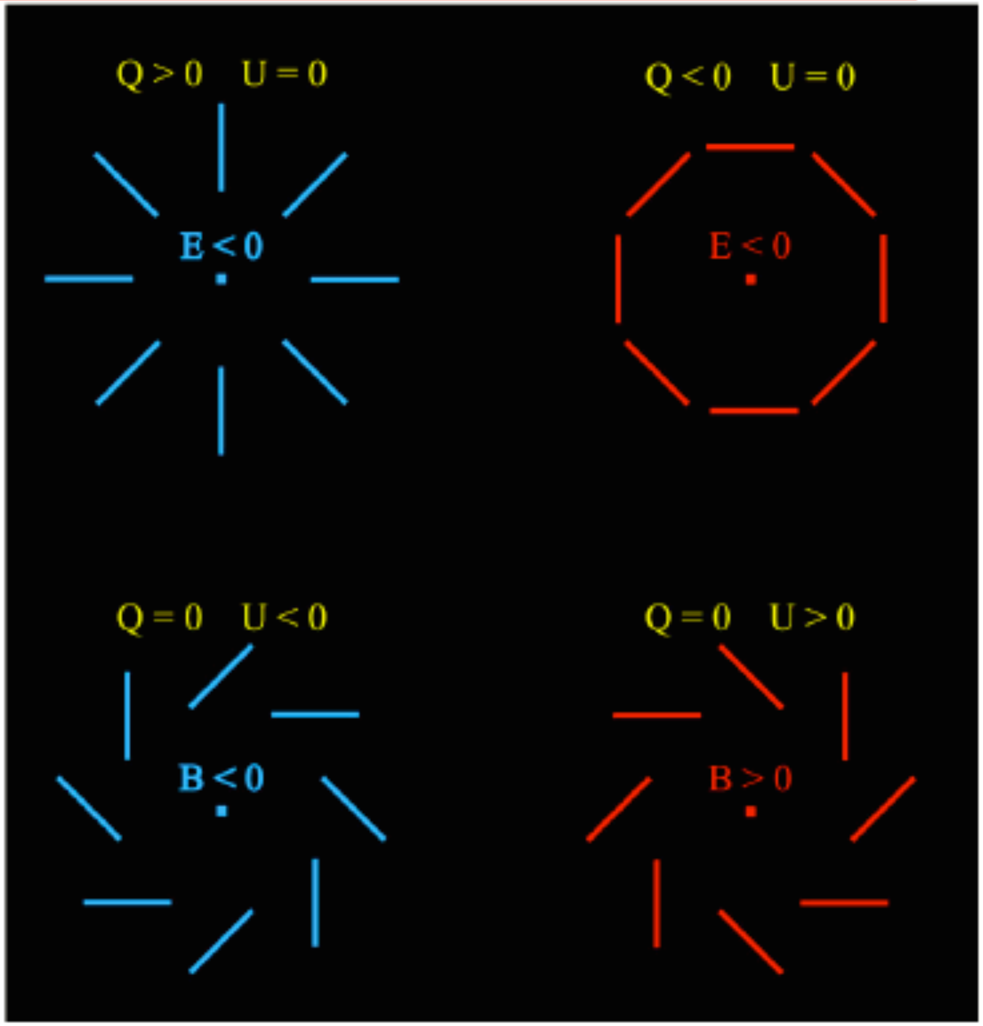
Scattering at z~1100 produces signal on degree scales

Scattering at z~10 produces signal on 10 degree scales - probes reionization from first stars.



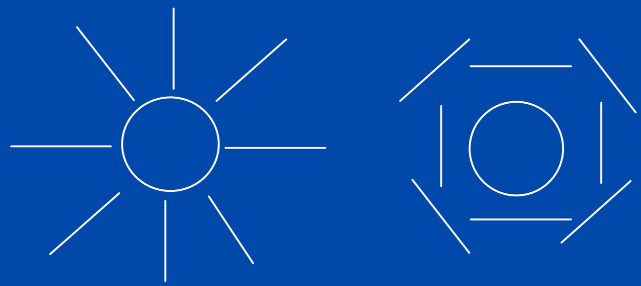
The E's and B's of Polarization Spectra

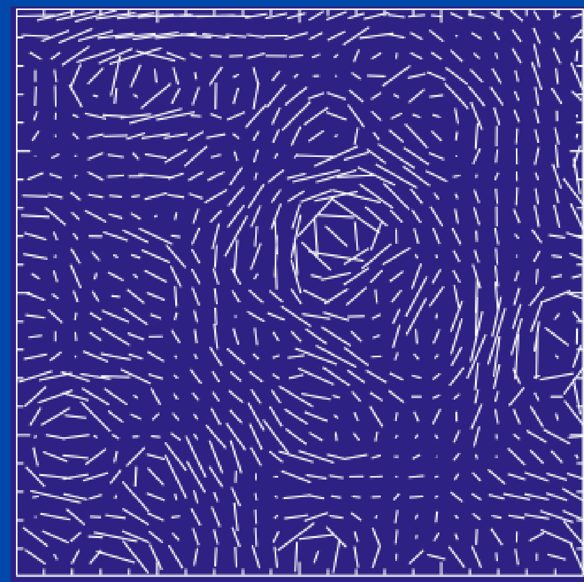
- Polarization decomposable into E mode (gradient) and B mode (curl) components.
- Tensor fluctuations produce both E and B mode components.
- Scalar fluctuations produce only E mode component (except for transformation by gravitational lensing).
- B modes directly probe gravity waves.



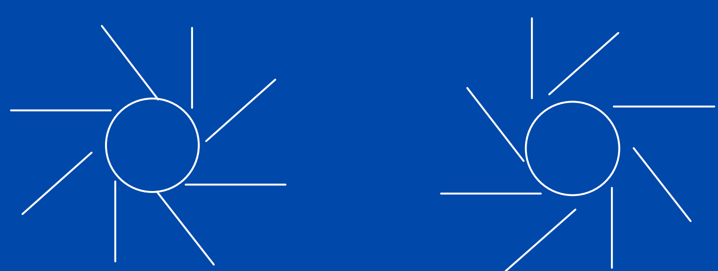
E and B modes polarization

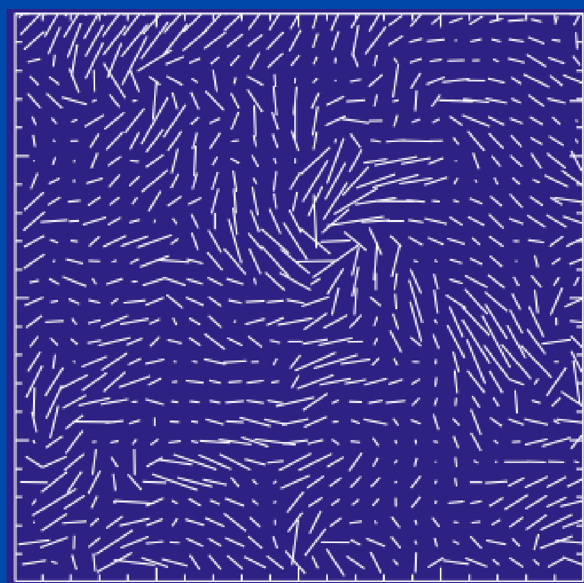
E polarization
from scalar, vector and tensor modes





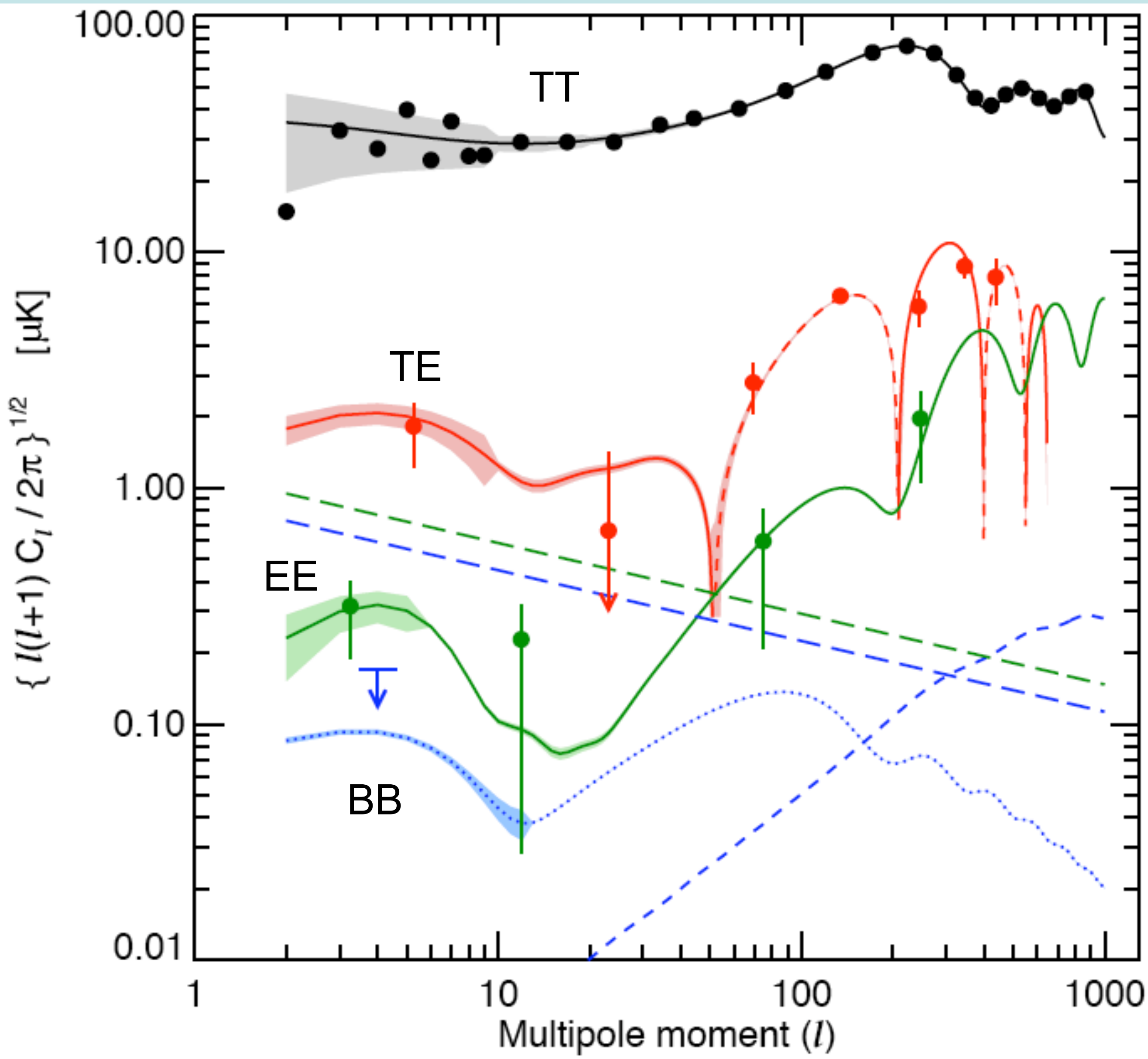
B polarization only from (vector)
tensor modes





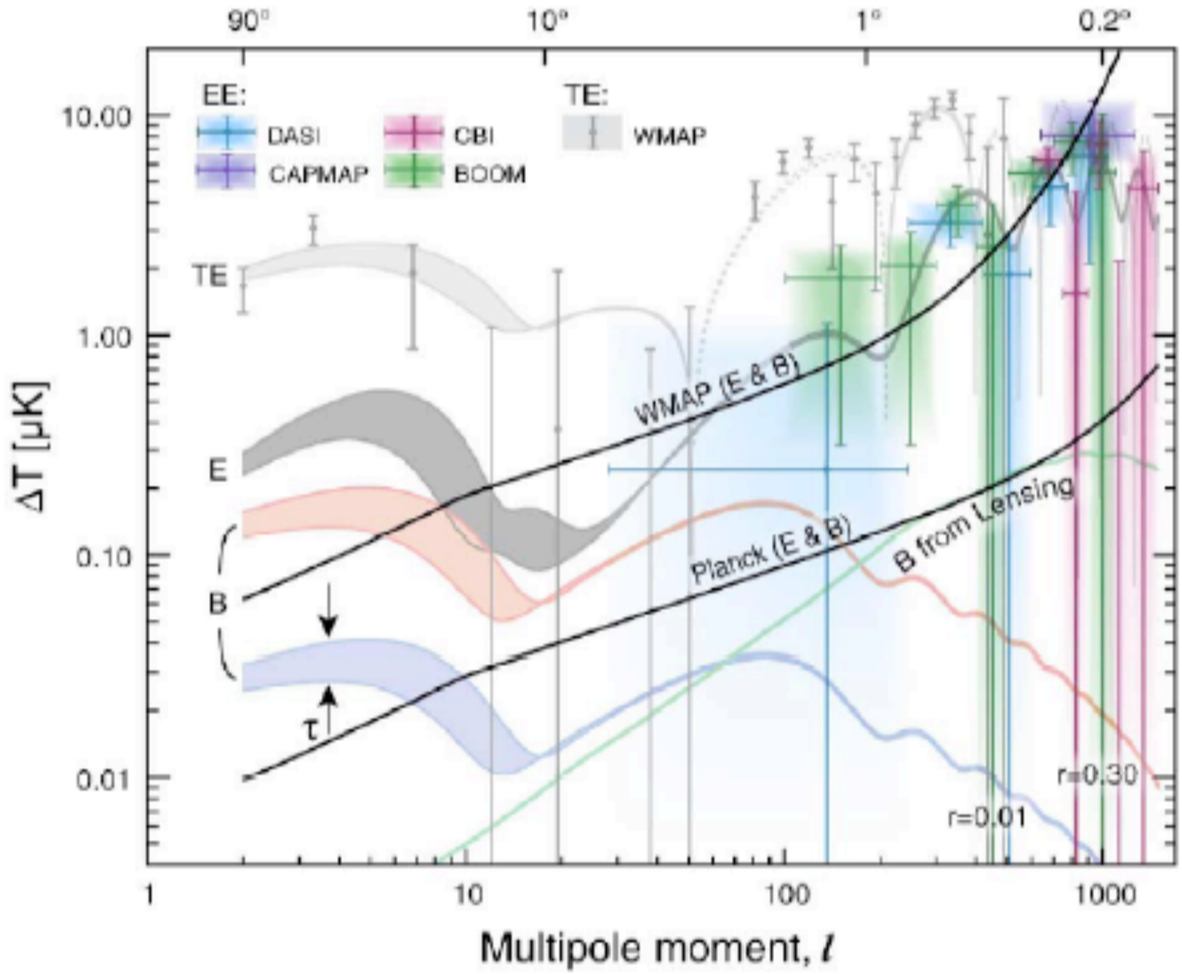
Kamionkowski, Kosowsky, Stebbings 1997, Zaldarriga & Seljak 1997

WMAP3 data

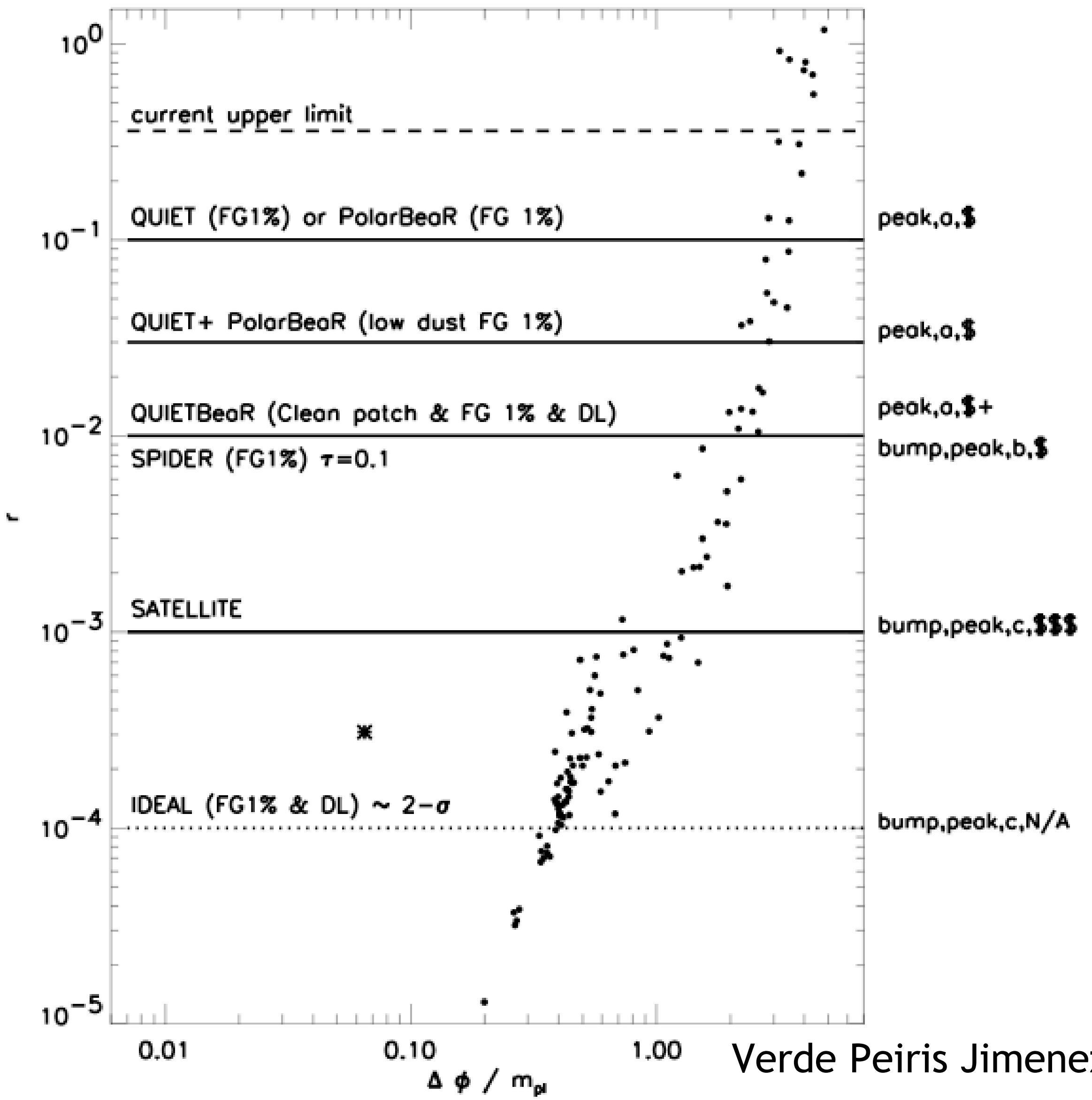


100.00
10.00
1.00
0.10
0.01
TT
TE
EE
BB
{ l(l+1) C_l / 2π }^1/2 [μK]
1
10
100
1000
Multipole moment (l)

The E's and B's of Polarization Spectra



Future prospects: gravity waves



Verde Peiris Jimenez 05

Tev

3.2×10^{13}

1.7×10^{13}

9.7×10^{12}

5.5×10^{12}

3×10^{12}

SUMMARY:

- I. The predictions of inflation are right:

  (i) the universe has a critical density

  (ii) Gaussian perturbations

  (iii) superhorizon fluctuations

  (iv) density perturbation spectrum nearly scale invariant

  (v) detection of polarization (from gravitational wave modes) in upcoming data may provide smoking gun for inflation

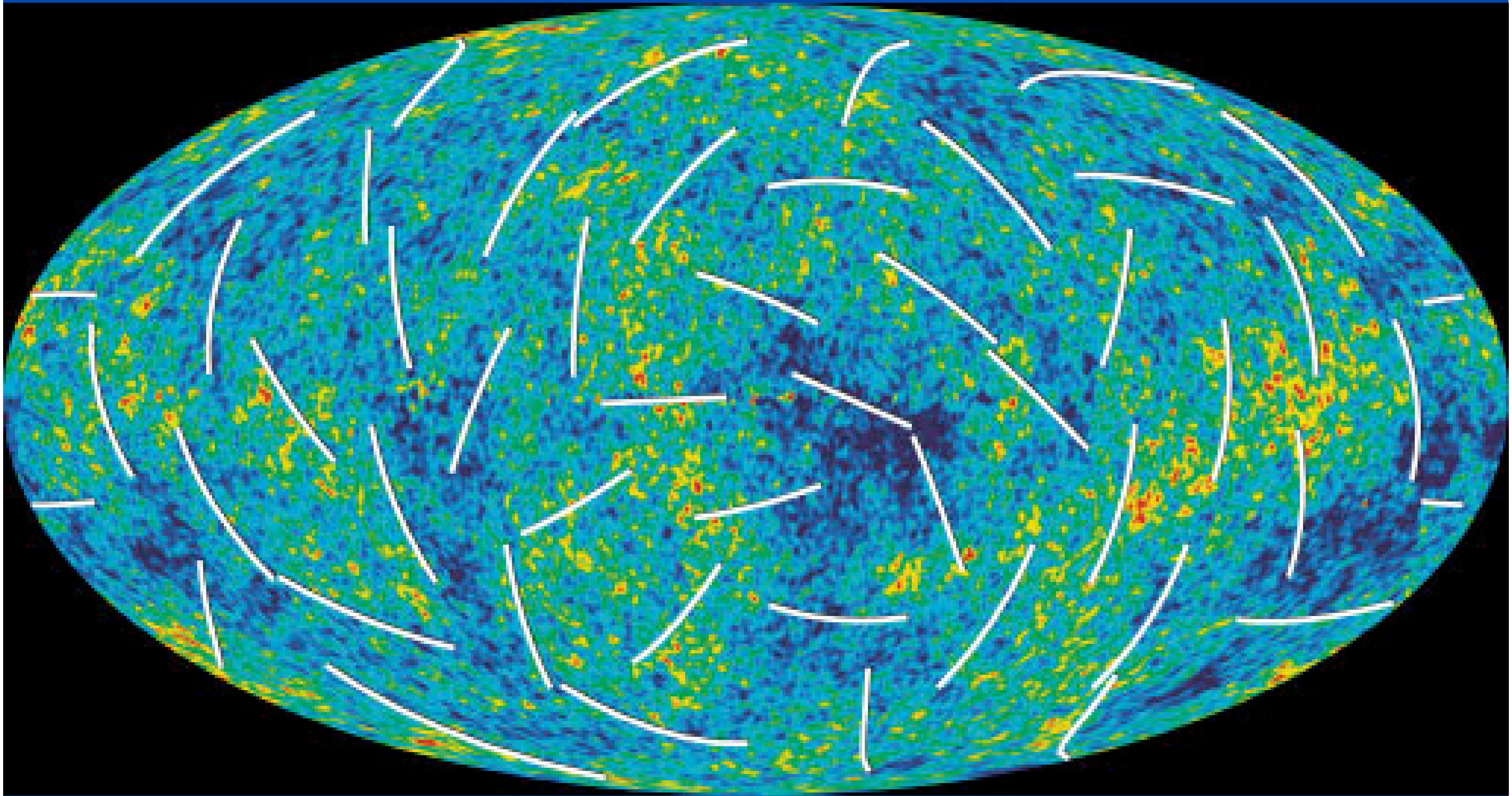
- II. Polarization measurements will tell us which model is right.

  WMAP already selects between models.

  Natural inflation (Freese, Frieman, Olinto) looks great

Predictions/Status of Inflation

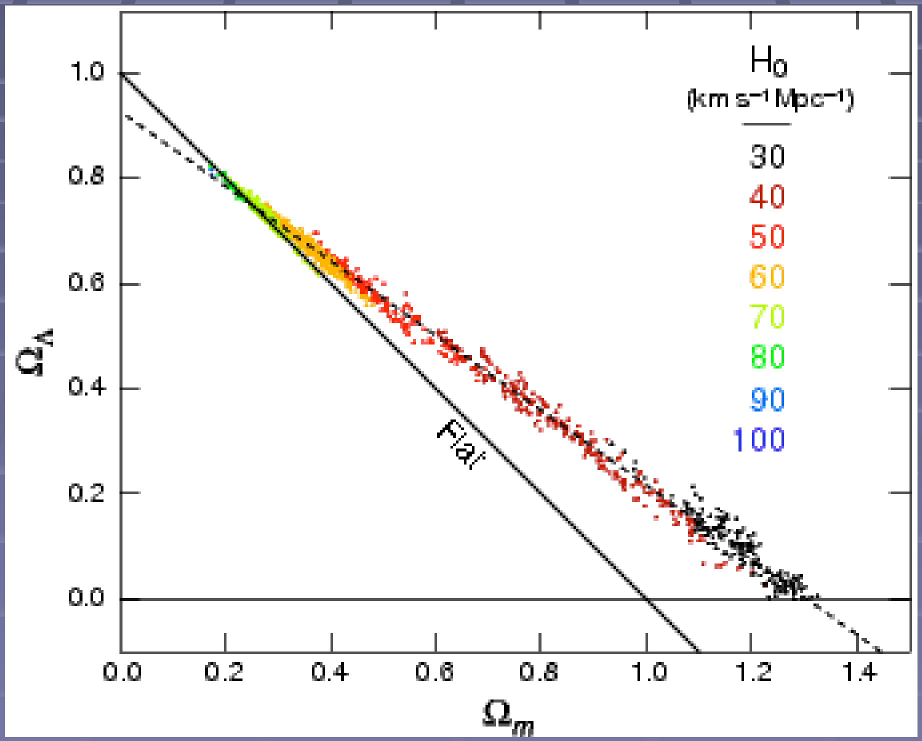
- The universe has a critical density √√
- Gaussian perturbations (single field models) √ (so far)
- Superhorizon fluctuations √
- Density perturbations $n_s \sim 1$ √
- Gravitation wave modes upcoming



end

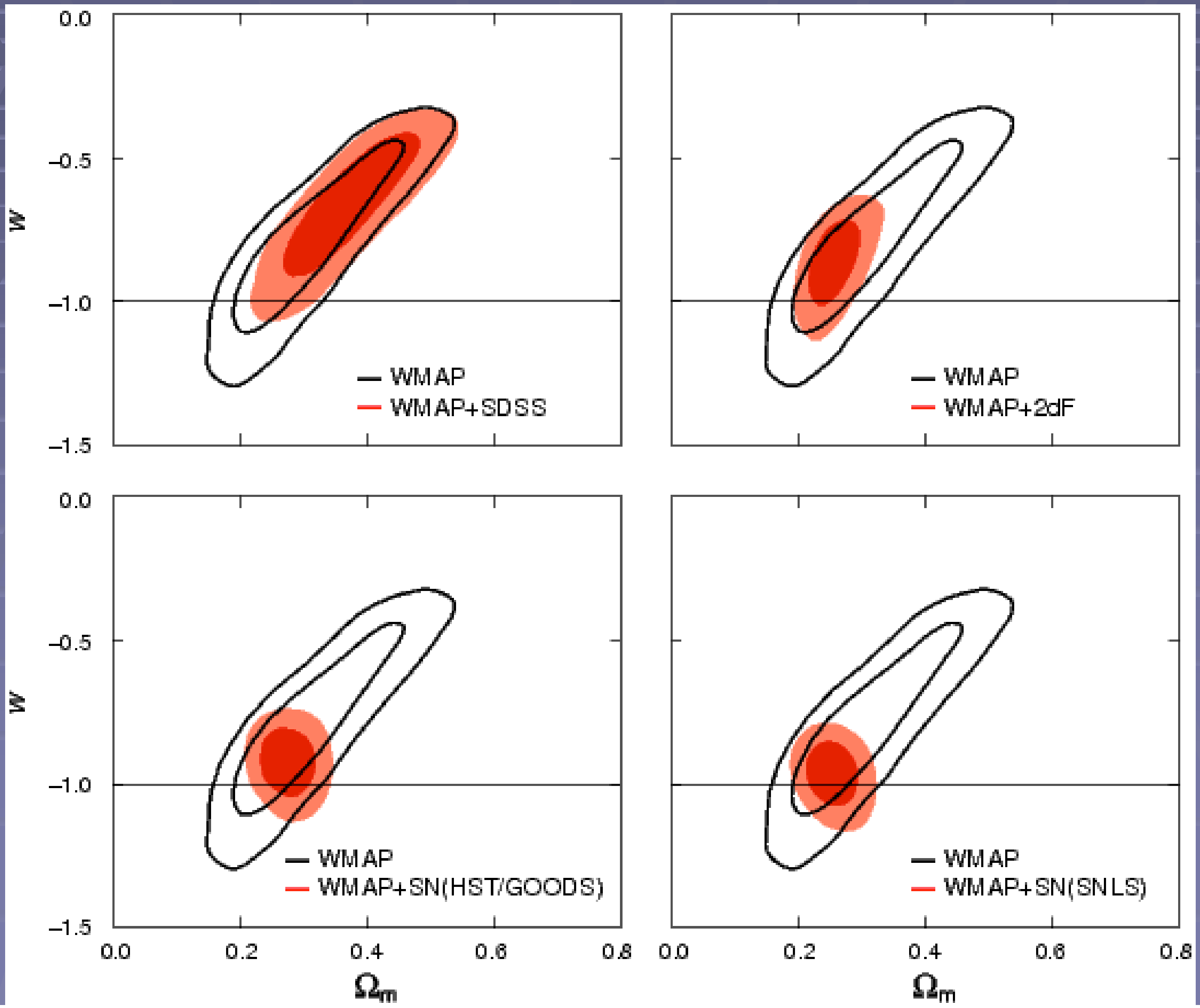
Tests of Inflation

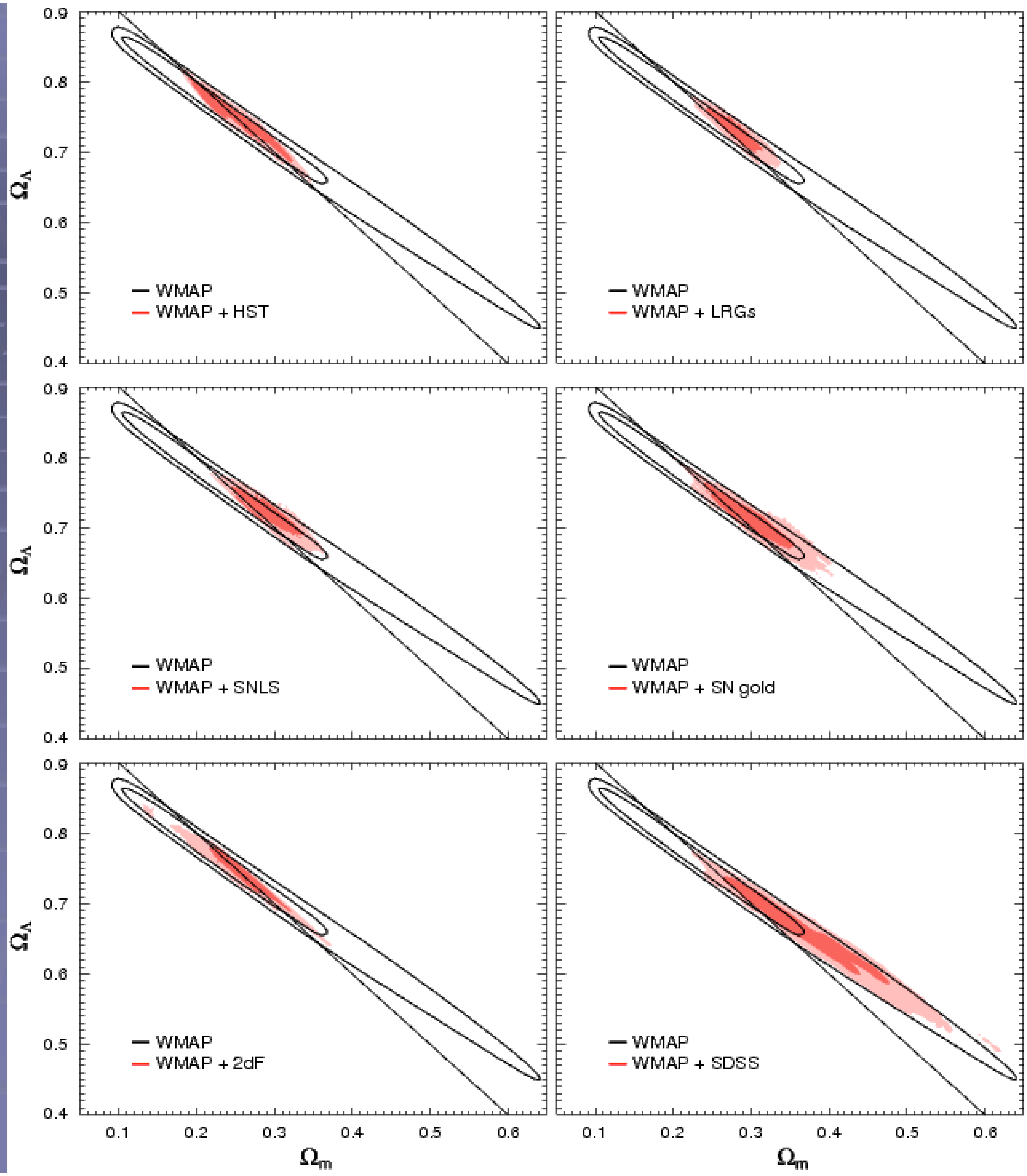
- The following are "generic" predictions of inflation, for which we had little evidence in the 1980's (adapted from Steinhardt):
  - near scale invariance [COBE]
  - flatness [TOCO, Boomerang,WMAP1]
  - adiabatic fluctuations [WMAP1]
  - gaussian fluctuations [WMAP1]
  - super-horizon fluctuations [WMAP1-TE]
  - ns < 1 [WMAP2, …]
  - gravity waves [CMBPOL?]



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Ω_Λ
Ω_m
Flat
H_0
(km s^{-1} Mpc^{-1})
30
40
50
60
70
80
90
100

DARK ENERGY (w=p/rho)





WMAP
WMAP + HST
WMAP
WMAP + LRGs
WMAP
WMAP + SNLS
WMAP
WMAP + SN gold
WMAP
WMAP + 2dF
WMAP
WMAP + SDSS
Ω_Λ
Ω_m

Testing Inflation: Gaussian, Random Phase?

>700 papers written since 1st data release based on WMAP results. Several questioning validity of standard model, specifically gaussianity of fluctuations.

Expand temperature field in Fourier space:

$$T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

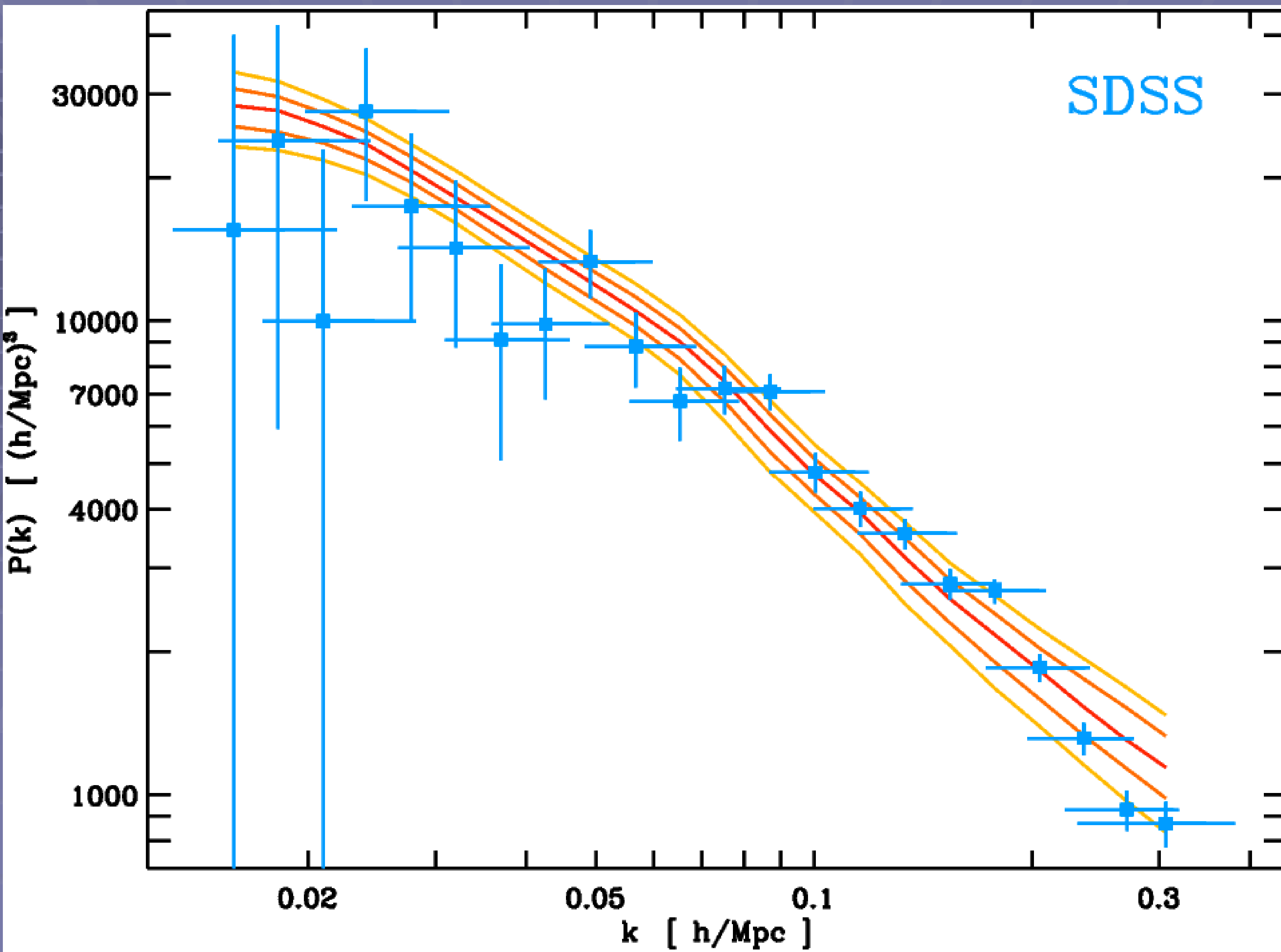
Simplest prediction of inflation is that a_{lm} coefficients are gaussian distributed:

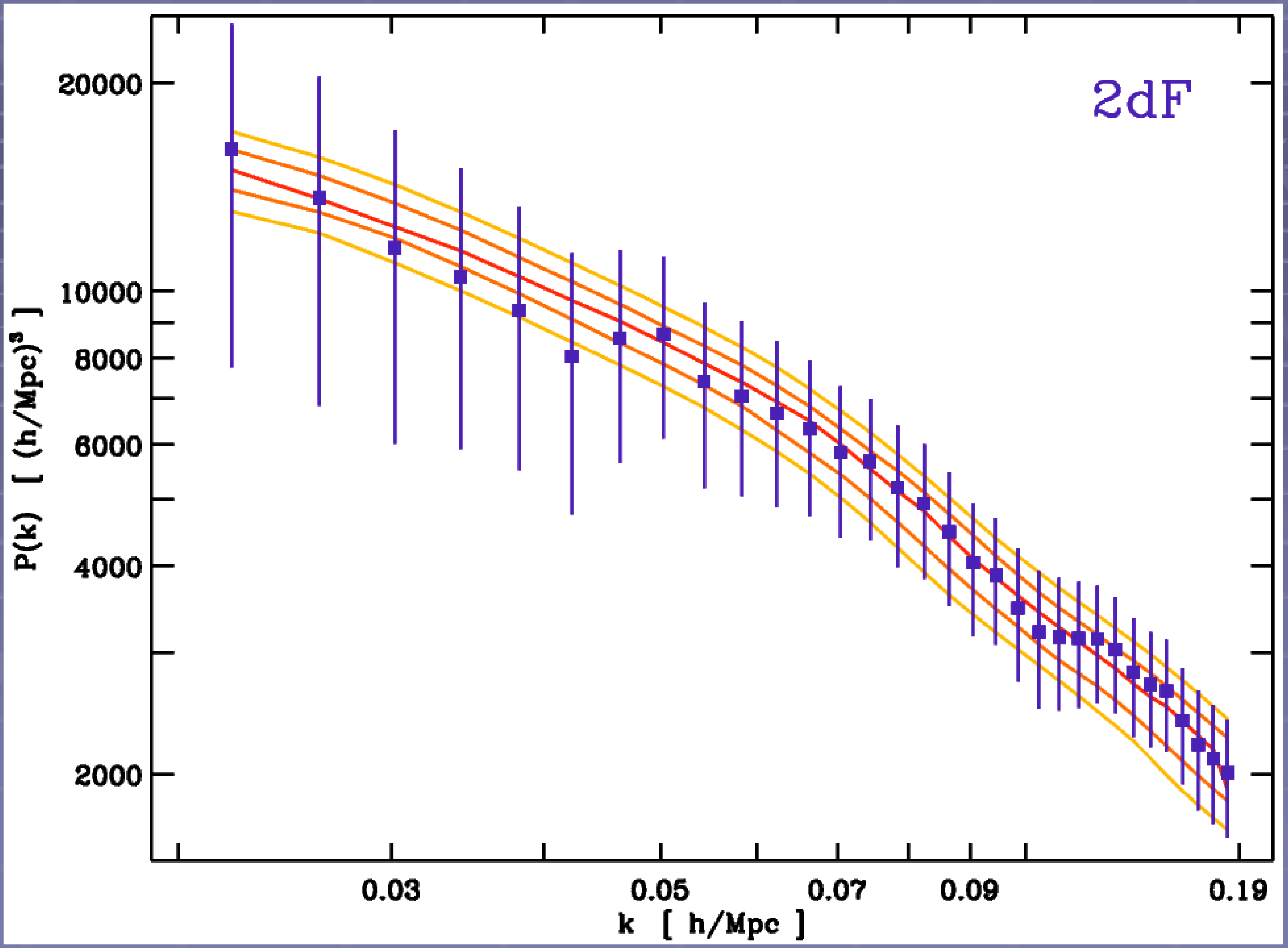
$$P(a_{lm}) \propto \exp(-\tfrac{1}{2} a_{lm}^2 / C_l)$$

with random (uncorrelated) phases:

$$\left\langle a_{lm} a_{l'm'}^* \right\rangle = C_l \delta_{ll'} \delta_{mm'}$$

STOP HERE


SDSS
30000
10000
7000
4000
1000
P(k) [(h/Mpc)³]
0.02
0.05
0.1
0.3
k [h/Mpc]



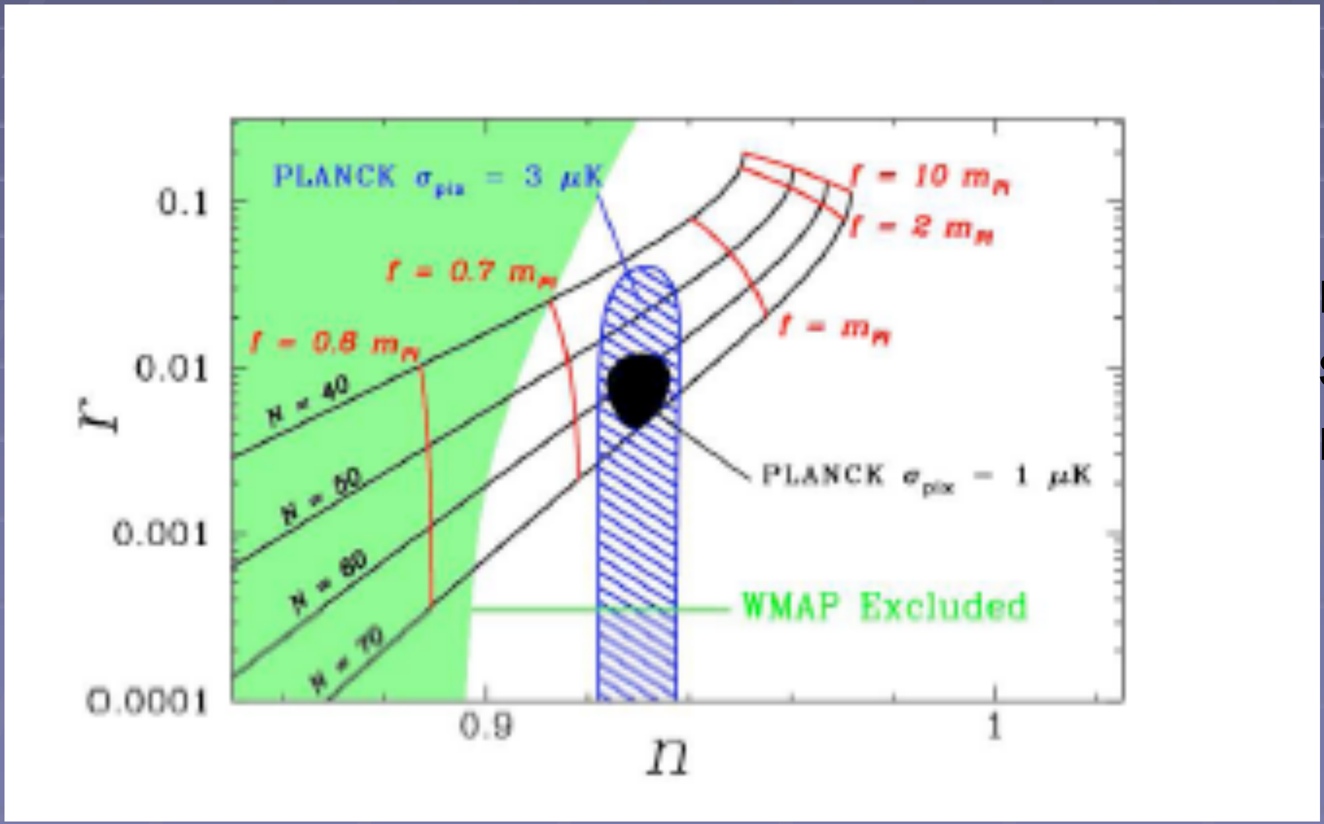
2dF
20000
10000
8000
6000
4000
2000
P(k) [(h/Mpc)³]
0.03
0.05
0.07
0.09
0.19
k [h/Mpc]

Prediction 2 of inflation is confirmed

- Multiple data sets (WMAP, large scale structure, etc) confirm n near 1.
- More detail shown in a minute to differentiate between models

Tensor-to-scalar ratio r vs. scalar spectral index n for natural inflation

(Freese and Kinney 2004)



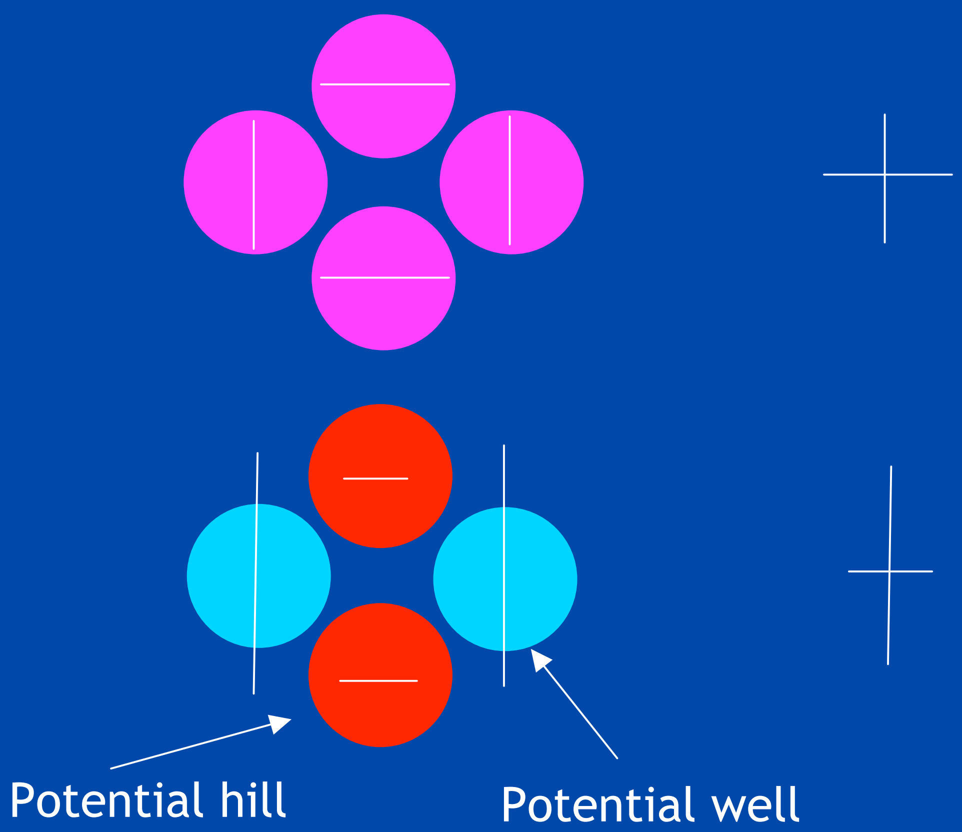
n.b. This is a small-field model

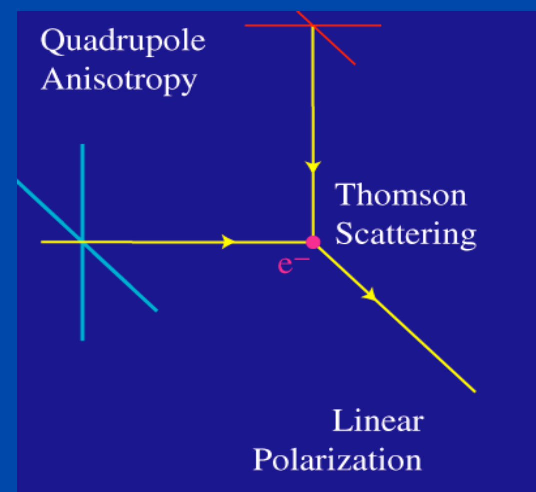
Conclusion

- An early period of inflation resolves cosmological puzzles: homogeneity, isotropy, oldness, and monopoles. It also generates density perturbations for galaxy formation.
- Details of density and gravitational wave modes can be used to test inflation as well as individual models.
- Predictions of inflation are confirmed!
- Natural inflation, which was theoretically well-motivated, fits the data very well.

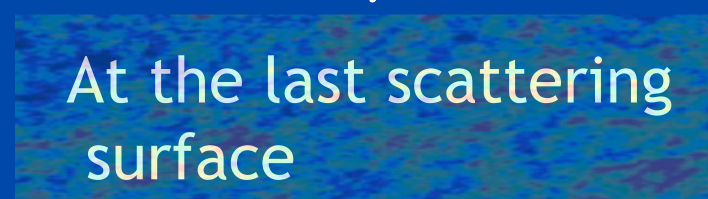
Generation of CMB polarization

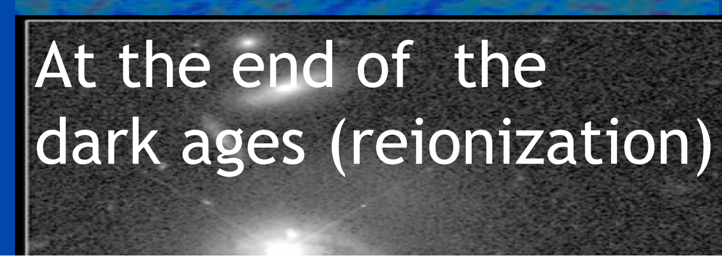
- Temperature quadrupole at the surface of last scatter generates polarization.





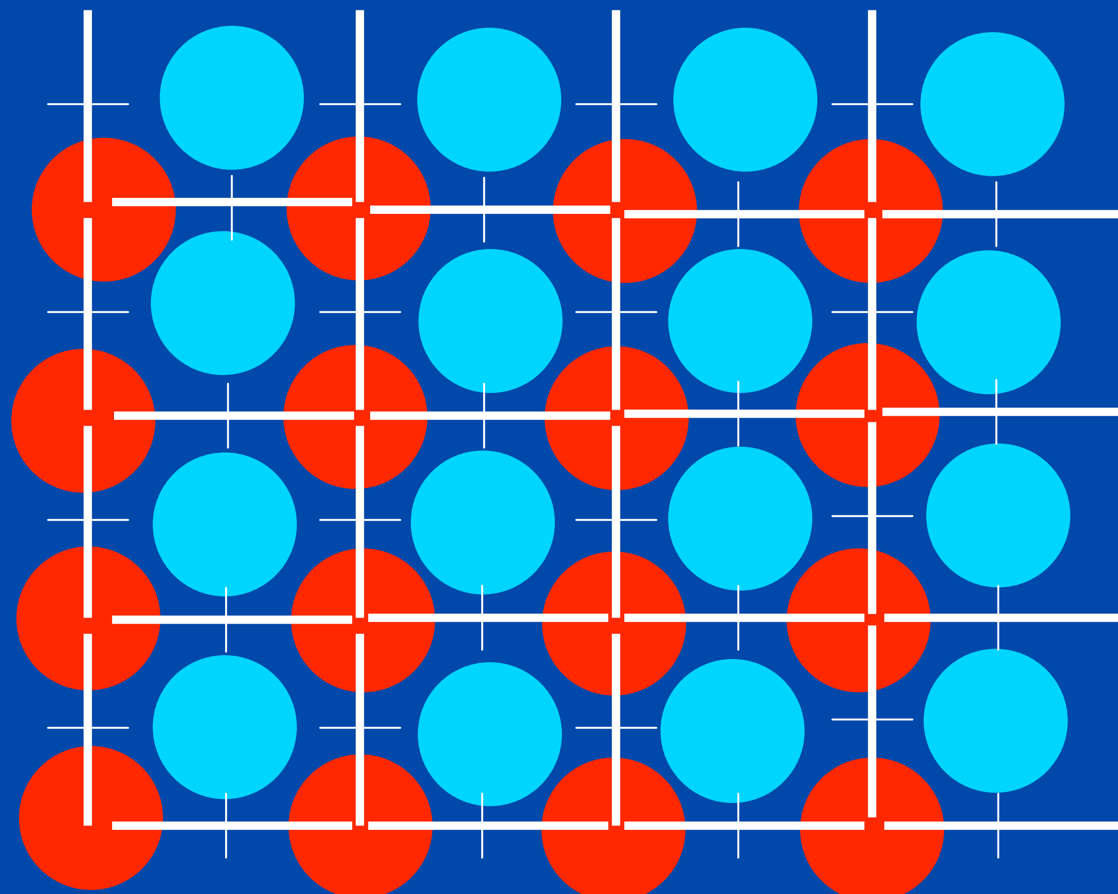
From Wayne Hu

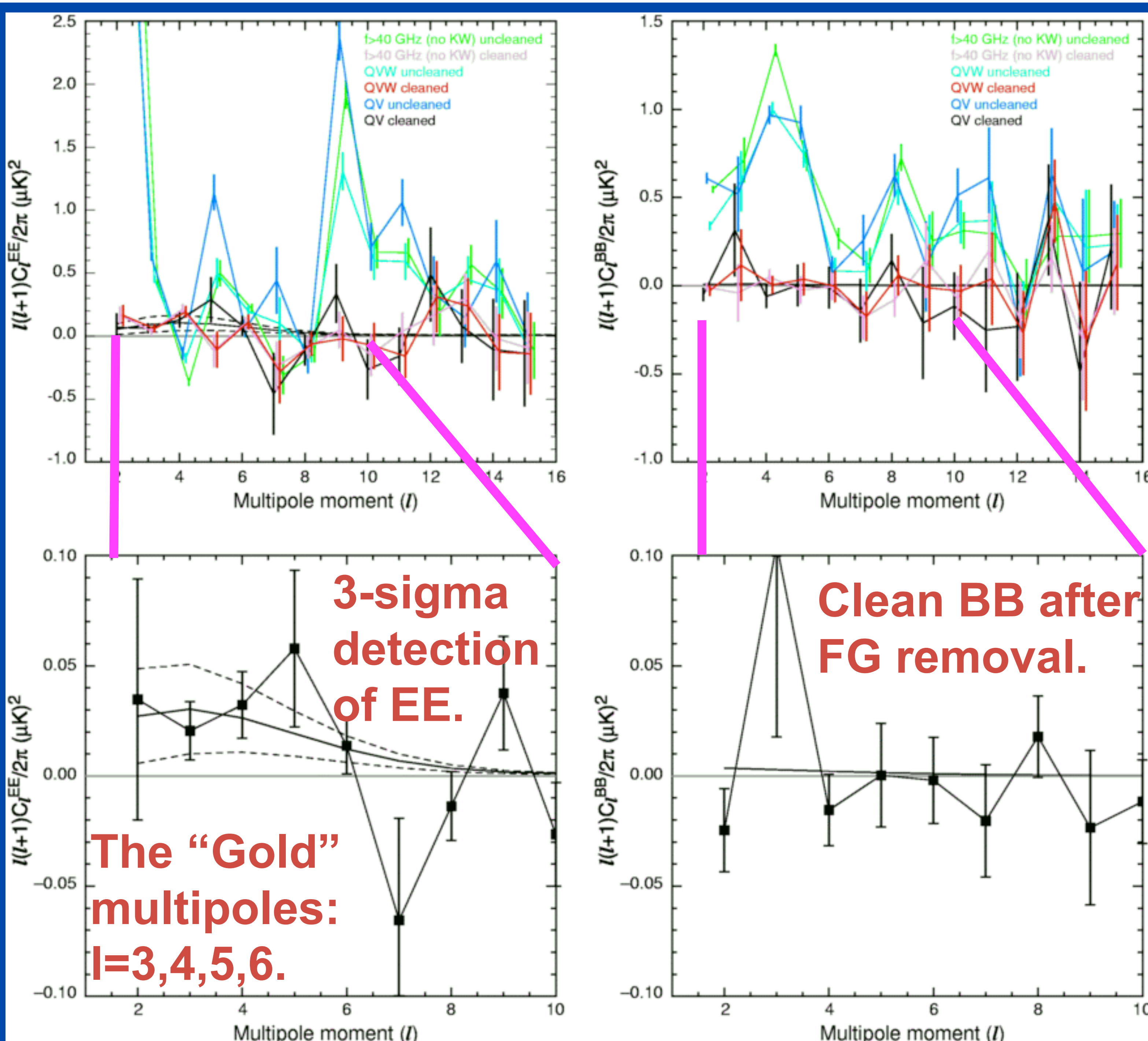




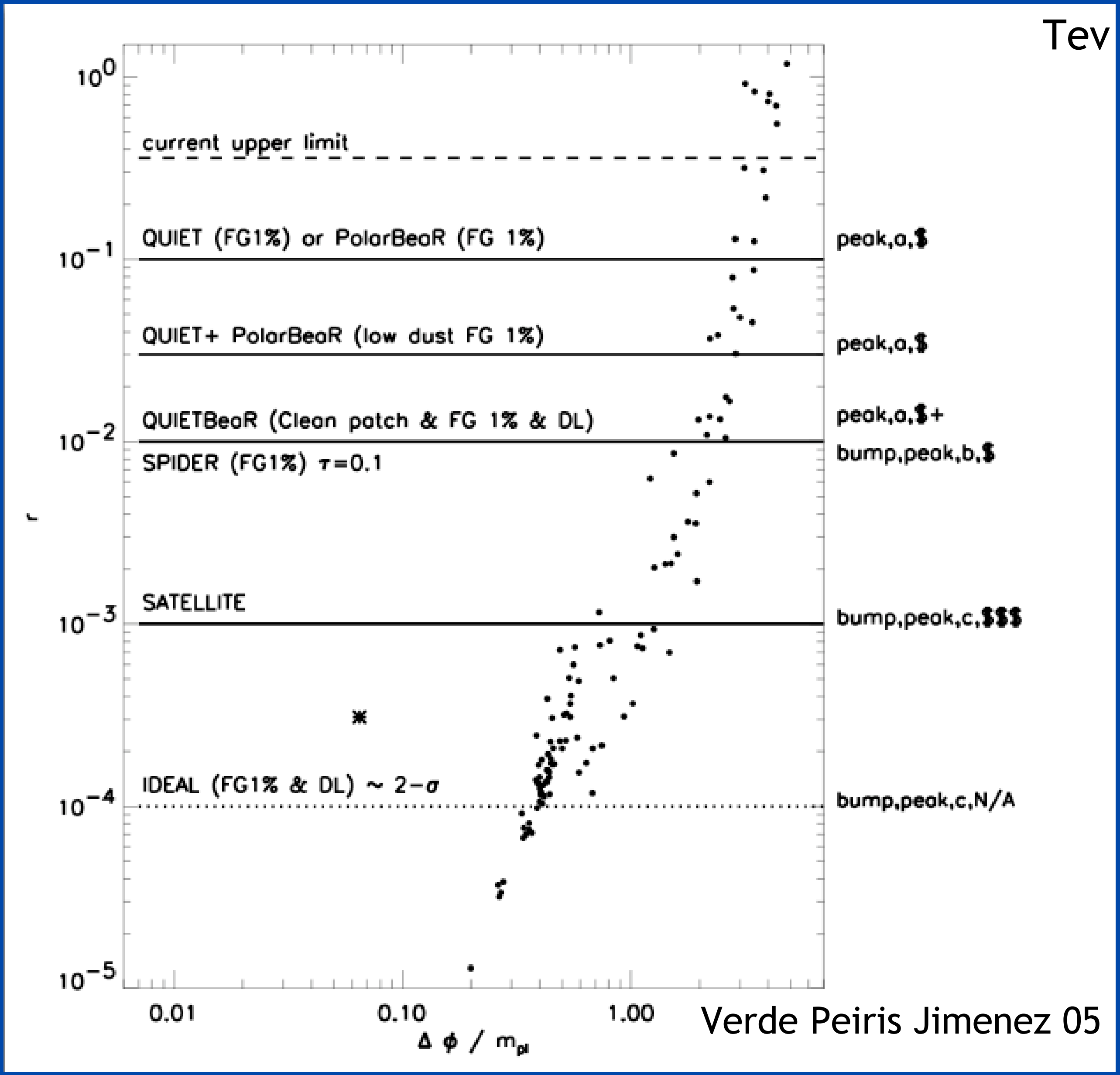
Polarization for density perturbation

- Radial (tangential) pattern around hot (cold) spots.




f>40 GHz (no KW) uncleaned
f>40 GHz (no KW) cleaned
QVW uncleaned
QVW cleaned
QV uncleaned
QV cleaned
$l(l+1)C_l^{EE}/2\pi$ (μK)^2
$l(l+1)C_l^{BB}/2\pi$ (μK)^2
Multipole moment (l)
3-sigma detection of EE.
The "Gold" multipoles: l=3,4,5,6.
Clean BB after FG removal.

Future prospects: gravity waves



3.2x10^13

1.7x10^13

9.7x10^12

5.5x10^12

3x10^12

Density Fluctuations and Tensor Modes can determine which model is right

- Density Fluctuations:

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}}, \quad |\delta_k|^2 \sim k^{n_s}$$

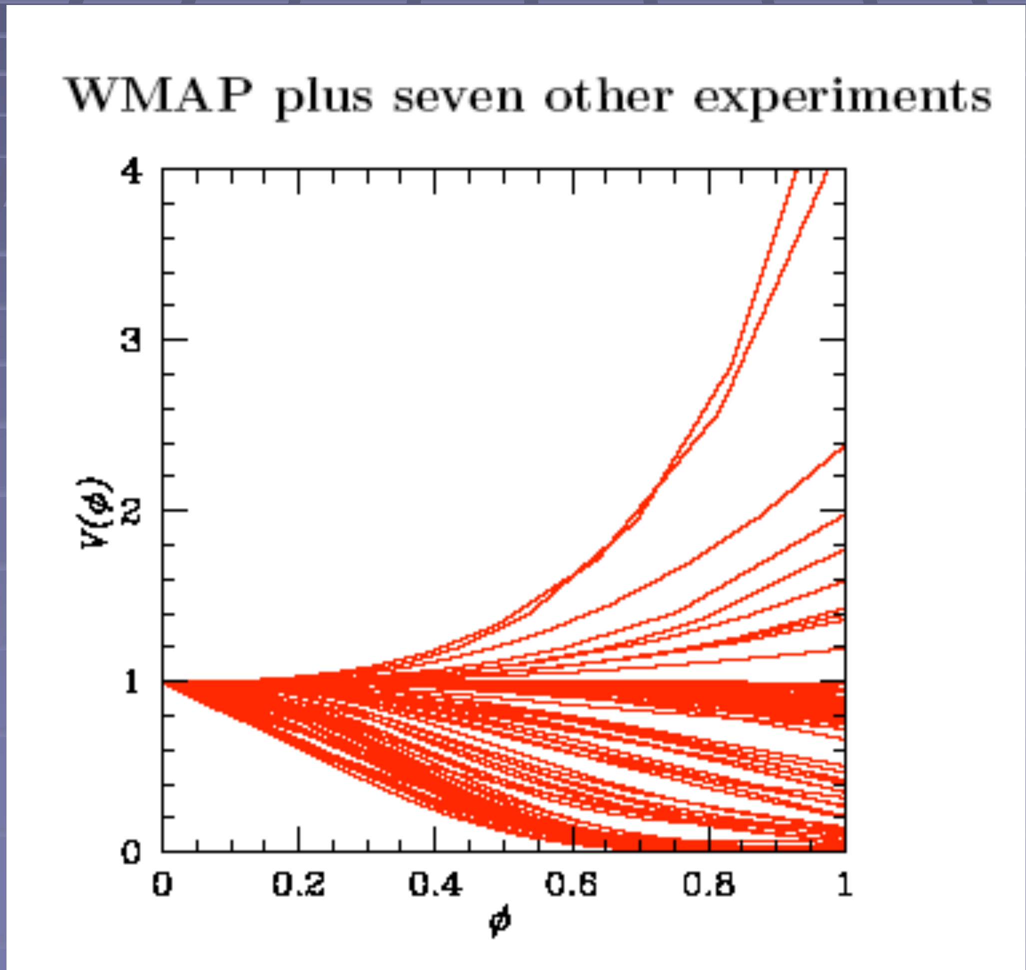
WMAP data:

$$|n_s - 1| < 0.1$$

Slight indication of running of spectral index

- Tensor Modes $$P_T^{1/2} = \frac{H}{2\pi}$$ gravitational wave modes, detectable in upcoming experiments

1 sigma reconstruction of potential from 1-year WMAP data



(KINNEY, KOLB, MELCHIORRI, AND RIOTTO 2003)

Present Horizon Scale

- What point during inflation corresponds to the horizon scale today?
- e-foldings <u>before</u> the end of inflation *N*: $a = a_e \, \mathrm{e}^{-N}$
- Depends on post-inflation physics

$$N(k) \approx 62 - \ln\frac{k}{a_0 H_0} - \ln\frac{10^{16}\,\mathrm{GeV}}{V_k^{1/4}} + \ln\frac{V_k^{1/4}}{V_e^{1/4}} - \frac{1}{3}\ln\frac{V_e^{1/4}}{\rho_{\mathrm{RH}}^{1/4}}$$

Potential when mode leaves horizon

Potential at end of inflation

energy density after reheating

Present Horizon Scale

- Typical $N \approx 60$
- Dodelson & Hui (2003) $N < 67$
- Liddle & Leach (2003) $50 < N < 60$
- Non-standard cosmologies
  - Non radiation-like period (post-reheat)

$$40 \leq N \leq 70$$

Slow Roll

Linde (1982)
Albrecht & Steinhardt (1982)

- Evolution of the field ($\Gamma \to 0$): $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

- Drop first term: $\ddot{\phi} << 3H\dot{\phi} \quad \text{for } \varepsilon, \eta << 1$

$$\varepsilon = \frac{M_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2 \approx \frac{1}{4\pi}\left(\frac{M_{Pl}}{f}\right)^2 \frac{\sin^2(\phi/f)}{[1+\cos(\phi/f)]^2}$$

$$\eta \equiv \frac{M_{Pl}^2}{4\pi}\left(\frac{H''(\phi)}{H(\phi)}\right) \approx -\frac{1}{16\pi}\left(\frac{M_{Pl}}{f}\right)^2$$

End of Inflation

- Inflation ends when field starts accelerating rapidly

$$\varepsilon = 1 \quad \Rightarrow \quad \phi_e \qquad\qquad \cos(\phi_e / f) = \frac{1 - 16\pi \left(\frac{f}{M_{Pl}}\right)^2}{1 + 16\pi \left(\frac{f}{M_{Pl}}\right)^2}$$

- Define field ϕ in terms of number of e-foldings N prior to the end of inflation, i.e. $\phi(N)$

$$\sin(\phi / 2f) = \sin(\phi_e / 2f) \exp\left[-\frac{1}{16\pi} \left(\frac{M_{Pl}}{f} \right)^2 N \right]$$